III YEAR - V SEMESTER
COURSE CODE: 7BMA5C3

CORE COURSE - XI – OPERATIONS RESEARCH - I

Unit – I

Introduction – Origin and Development of O.R – Nature and features of O.R. – Scientific Method in O.R. – Modelling in O.R. – Advantages and Limitations of Models – General solution methods of O.R. models – Applications of Operations Research – Linear Programming problem – Mathematical formulation of the problem – Illustration on Mathematical formulation of linear programming problems – Graphical solution method – Some exceptional cases – General linear programming problem – Canonical and Standard forms of L.P.P – Simplex method.

Unit – II

Use of Artificial variables (Big M method – Two Phase method) Duality in linear programming – General primal and dual pair – Formulating a Dual problem – Primal – Dual pair in matrix form – Duality Theorems – Complementary Slackness Theorem – Duality and Simplex method – Dual simplex method.

Unit – III

Introduction – L.P. formulation of T.P. – Existence of solution in T.P. – The Transportation table – Loops in T.P. – Solution of a Transportation problem – Finding an initial basic – feasible solution (NWCM – LCM – VAM) – Degeneracy in TP – Transportation Algorithm (MODI Method) – Unbalanced T.P – Maximization T.P.

Unit – IV

Assignment problem – Introduction – Mathematical formulation of the problem – Test for optimality by using Hungarian method – Maximization case in Assignment problem.

Unit – V

Sequencing problem – Introduction – problem of sequencing – Basic terms used in Sequencing– n jobs to be operated on two machines – problems – n jobs to be operated on K machines–problems–Two jobs to be operated on K machines (Graphical method)–problems.

Text Book:

1. Operations Research (14th edition) by KantiSwarup, P.K.Gupta and Man Mohan, Sultan Chand & Sons, New Delhi, 2008.

Unit I	Chapter 1sections 1.1 to 1.7, 1.10 Chapter 2 sections 2.1 to 2.4 Chapter 3 sections 3.1 to 3.5 Chapter 4 sections 4.1 to 4.3
Unit II	Chapter 4 sections 4.4 Chapter 5 sections 5.1 to 5.7, 5.9
Unit III	Chapter 10 sections 10.1 to 10.3, 10.5, 10.6, 10.8, 10.9, 10.12, 10.13, 10.15
Unit IV	Chapter 11 sections 11.1 to 11.4
Unit V	Chapter 12 sections 12.1 to 12.6

Books for Reference:

1. P.K.Gupta and D.S.Hira, Operations Research, 2nd Edition, S.Chand& Co., New Delhi, 2004.
2. Taha H.A.,Operations Research–An Introduction.8th edition,Pearson Prentice Hall.

* * *

Linear Programming problem

mathematical formulation of the problem.

step 1 :-

study the given situation to find the key decisions to be made.

step 2 :-

Identify the various involved and designate them by symbols, x_j $(j = 1, 2, 3 \ldots)$

step 3 :-

state the feasible alternatives which generally are $x_j \geq 0$, For all j.

step 4 :-

Identify the constraints in the problem and express them as Linear inequalities For equations, LHS OF which are Linear. Functions OF the decision variables.

step 5 :-

Identify the objective Function and express it as a Linear Function OF the decision variables.

Graphical solution method.

301) A company makes two kinds of leather belts Belt A is a high quality belt & belt B is of Lower quality. The respective profits of Rs. 4 and Rs. 5 per but Each belt of type A requires twice as much time as a belt of type B and if all belts of type B the company could make 1000 belts per day the supply of leather is sufficient for only 800 belts per day (Both A&B) Belt A requires a fancy buckle and only 400 buckles per day are available. There are only 700 buckles a day available for belt B. Determine the optimal product min.

$$\max z = 4x_1 + 5x_2$$

subject to

$$2x_1 + x_2 \le 1000$$

$$x_1 + x_2 \le 800$$

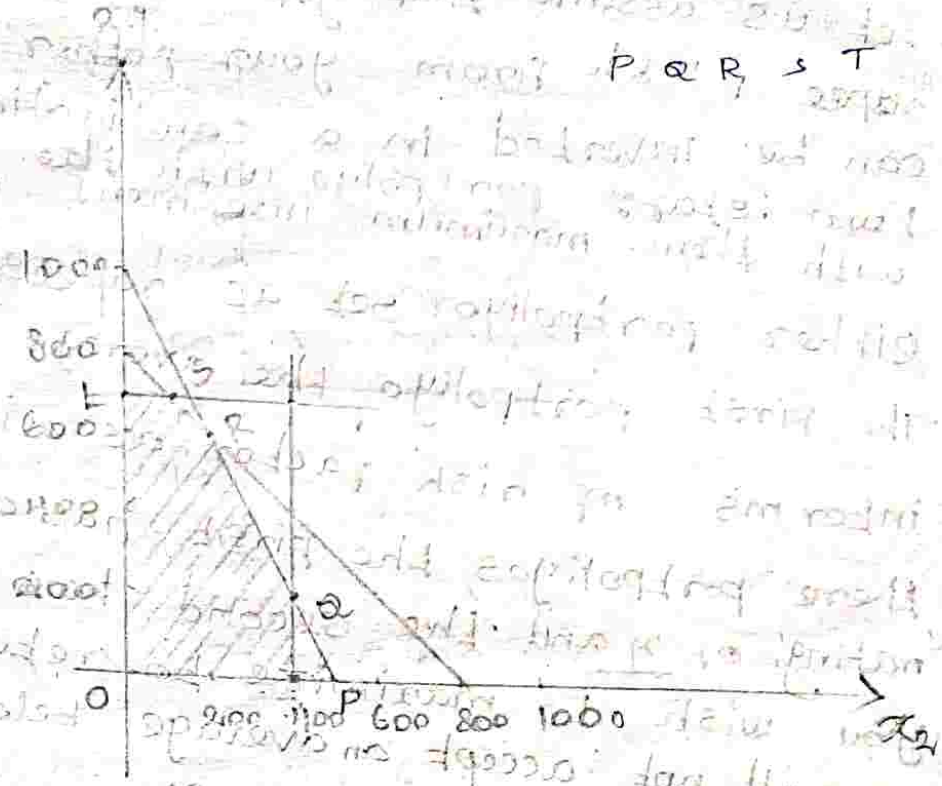
A) $x_1 \le 400$

B) $x_2 \le 700$

$$\frac{x_1}{800} + \frac{x_2}{800} \le 1$$

$(x_1, x_2 \ge 0)$

$$\frac{\cancel{2}x_1}{\overset{\cancel{1000}}{500}} + \frac{x_2}{1000} \le 1$$



point	(x_1, x_2)	$Z = 4x_1 + 3x_2$
O	(0,0)	0
P	(400,0)	1600
Q	(400,200)	2200
R	(200,600)	2600
S	(100,700)	2500
T	(0,700)	2100

a(0)+3(400)
= 1200

4(200)+
3(600)

800+1800

max Z = 2600

$x_p = 200$

$x_q = 600$

302. Let us assume that you have inharited
rupee 1 lakh from your father in Law that
can be inverted in a combination of only
two .staify portpolyo ~~with the~~ allowed in
with them maximum invasment
either portpoliyo set as rupees T5,000
The first portpoliyo the average rate
interms of risk factor associated with
there portpoliyos the first has a risk
rating of 4 and the second has a since
you wish to maximize to return you
we will not accept an average belong rate
a risk factor above Hence
How much shoult

object maximize $0,10 \cdot x_1$ $0.20 = x_2$

$max z = 0,10x_1 + 0.20x_2$

$= x_1 + x_2 = 100000$

$x_1 \leq T5000$

$x_2 \leq T5000$

average $= 0.12x_1 + 0.12x_2$

$= 0.0.02x_1 + 0.08x_2 \geq 0$

$0.10 + 0.20 \geq 0.2(x_1 + x_2)$

$4x_1 + 9x_2 \leq 6(x_1 + x_2)$

$4x_1 - 6x_1 + 9x_2 - 6x_2 \leq 0$

$-2x_1 + 5x_2 \leq 0$

$0.10x_1 + 0.20x_2 \geq 0.12(x_1 + x_2)$

$-2x_1 + 3x_2 \le 0$

$-2(25000) + 3x_2$

$$\begin{array}{r} 25000 \\ 2 \\ \hline 50000 \end{array}$$

$+150000 + 3x_2$

$x_2 = \frac{50000}{3} = 16.666.$

$-2(50000) + 3x_2$

$-100,000 + 3x_2$

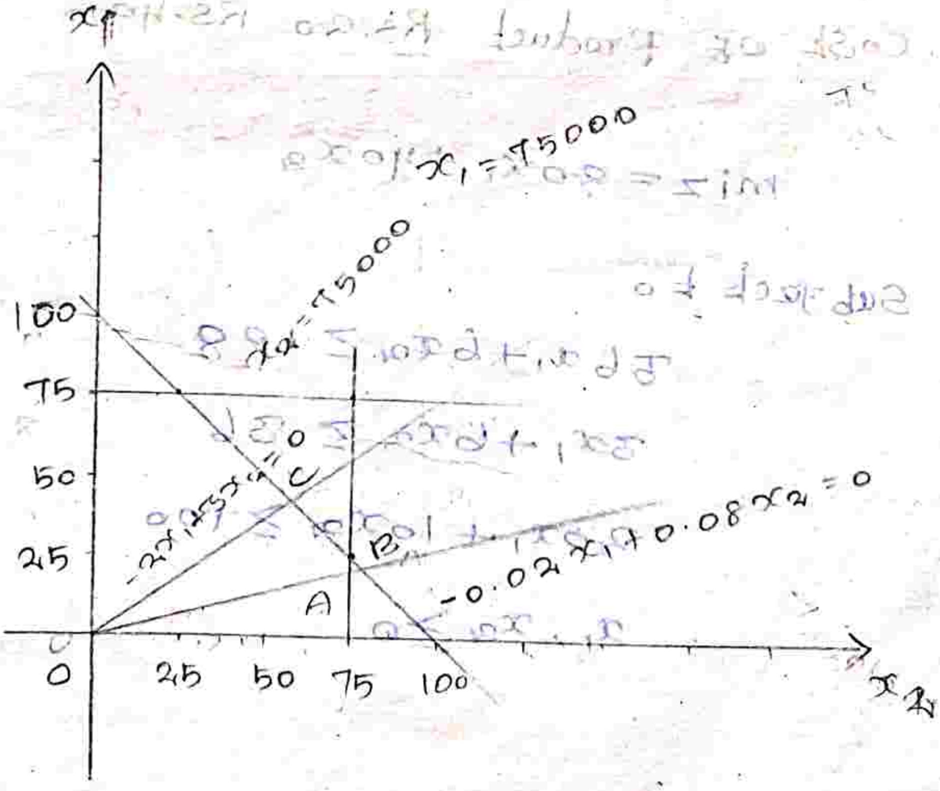
$x_2 = \frac{-100000}{3} = 33,333$

$-2(75000) + 3x_2$

$x_2 = \frac{150000}{3} = 50,000$

$-2(100000) + 3x_2$

$x_2 = \frac{200000}{3} = 66.666.$



Point	(x_1, x_2)	$Z = 0.10x_1 + 0.20x_2$
O	(0, 0)	0
A	(75000, 18750)	11250
B	(75000, 25000)	12500
C	(60,000, 40000)	14000

max $Z = 14000$

$x_1 = 60000$

$x_2 = 40000$

उ03. A Form is engaged in.

Nutrient Constents	Nutrient content in product		minimum amount of nutrient
	A	B	
X	36	06	108
Y	03	12	36
Z	20	10	100
Cost of product	Rs.20	Rs.40	

$$\min Z = 20x_1 + 40x_2$$

Subject to

$$36x_1 + 6x_2 \geq 108$$

$$3x_1 + 6x_2 \geq 36$$

$$20x_1 + 10x_2 \geq 100$$

$$x_1, x_2 \geq 0$$

$\frac{36}{108}x_1 + \frac{6}{108}x_2 =$

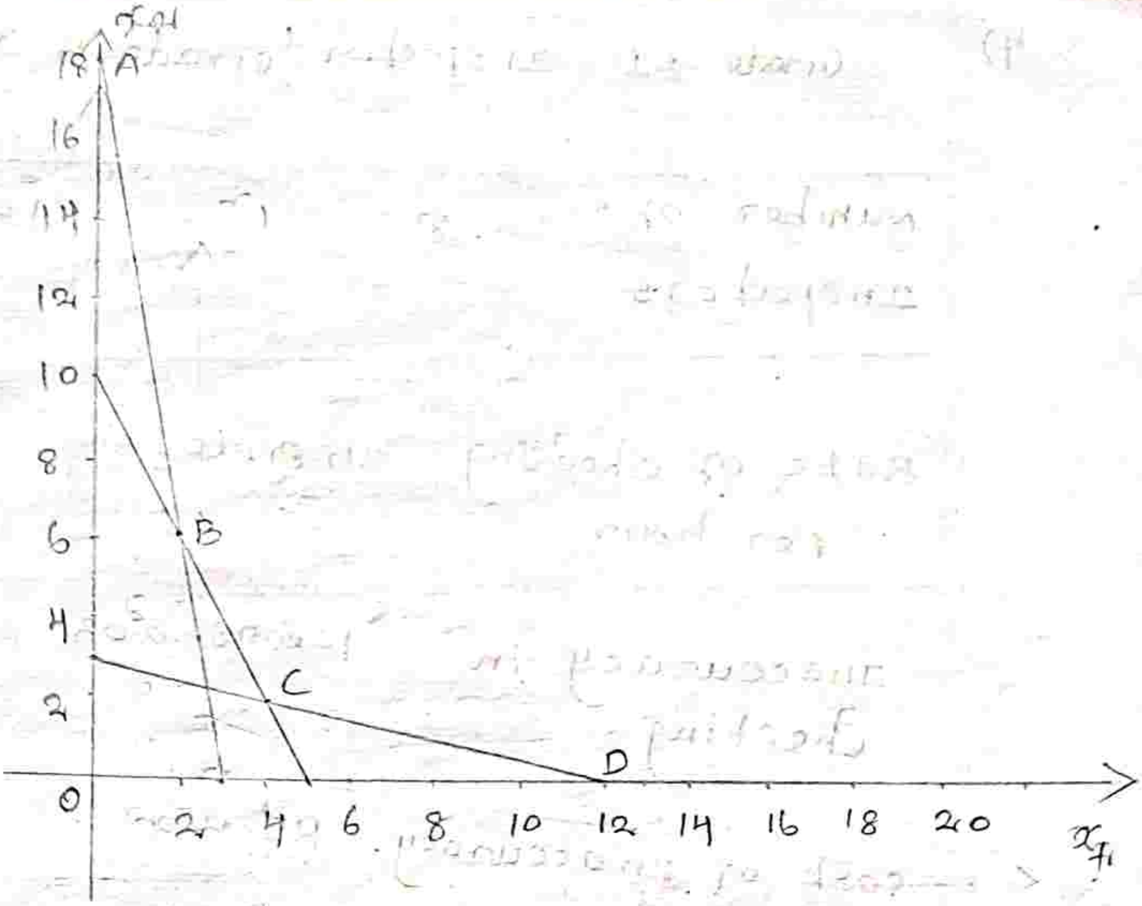
$\frac{x_1}{3} + \frac{x_2}{18}$

$\frac{3}{36} + \frac{12}{36}$

$x_1/12 + x_2/3$

$\frac{20}{100} + \frac{10}{100}$

$x_1/5 + \frac{x_2}{10}$



point	(x_1, x_2)	$+20x_1+40x_2$
A	(0, 18)	720
B	(2, 6)	280
C	(4, 2)	160
D	(12, 0)	240

$\min z = 160$

$x_1 = 4$

$x_2 = 2$

304)

	Grade · 1 Inspector	Grade 2 Inspector
Number of Inspectors	8	10
Rate of checking per hour	25 pieces	15 pieces
Inaccuracy in checking	1-0.98=0.02	1-0.95=0.05
cost of Inaccuracy in checking	RS. 20	RS.20
wage rate perhour	RS.40	RS.30

Grade 1 $= RS(40+20\times0.02\times25) = RS.50$

Grade 2 $= RS(30+20\times0.05\times15) = RS.45$

$\min z = 8\times50x_1 + 8\times4.5x_2$

$\min z = 400x_1 + 360x_2$

$8\times25x_1 + 8\times15x_2 \geq 1800$

(or)

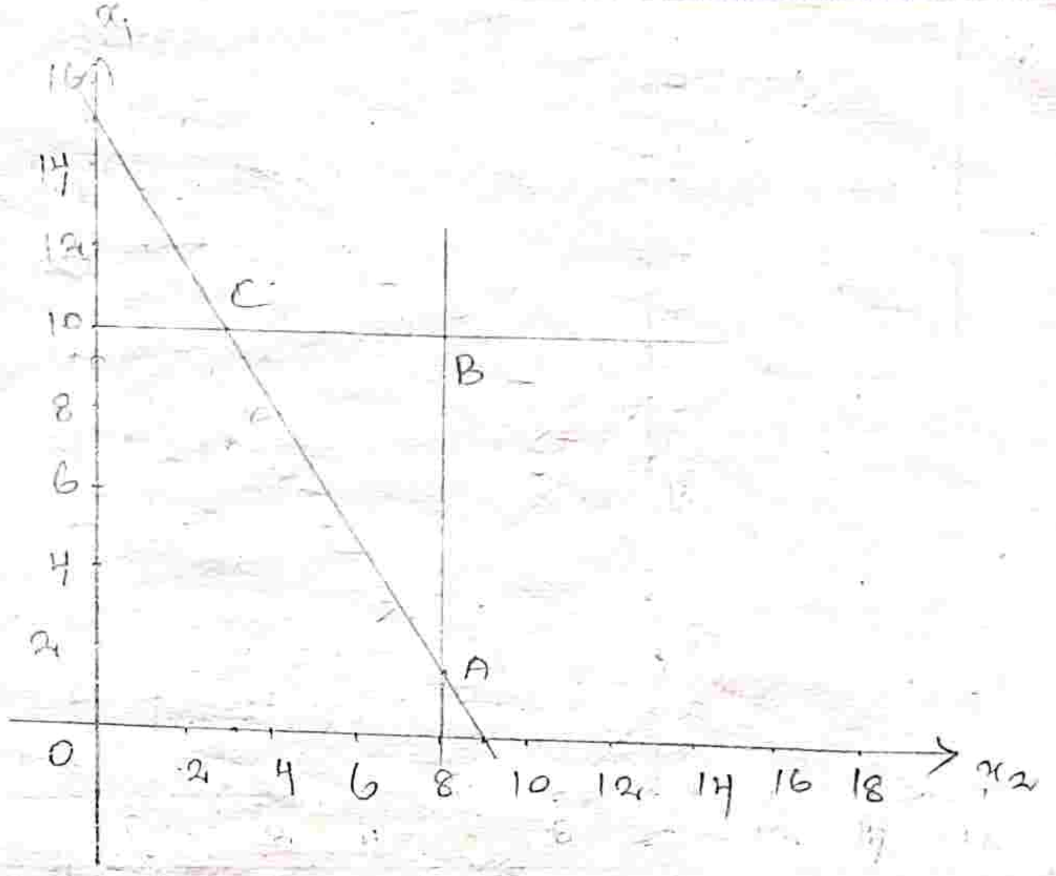
$5x_1 + 3x_2 \geq 45$

$x_1 \leq 8,\ x_2 \leq 10$

$x_1, x_2 \geq 0$

$\frac{5x_1}{\cancel{45}} + \frac{\overset{1}{\cancel{3}}x_2}{\underset{15}{\cancel{45}}} = 45$

$\frac{x_1}{9} + \frac{x_2}{15}$



Point	$(x_1 \ x_2)$	Z
A	$(8, 5/3)$	3800
B	$(8, 10)$	6800
C	$(3, 10)$	4800

$400(x_1) + 360x_2$

$400(8) + 360(5/3)$

$3200 + 600$

$3800.$

$\min z = 3800$

$x_1 = 8,$

$x_2 = 5/3$

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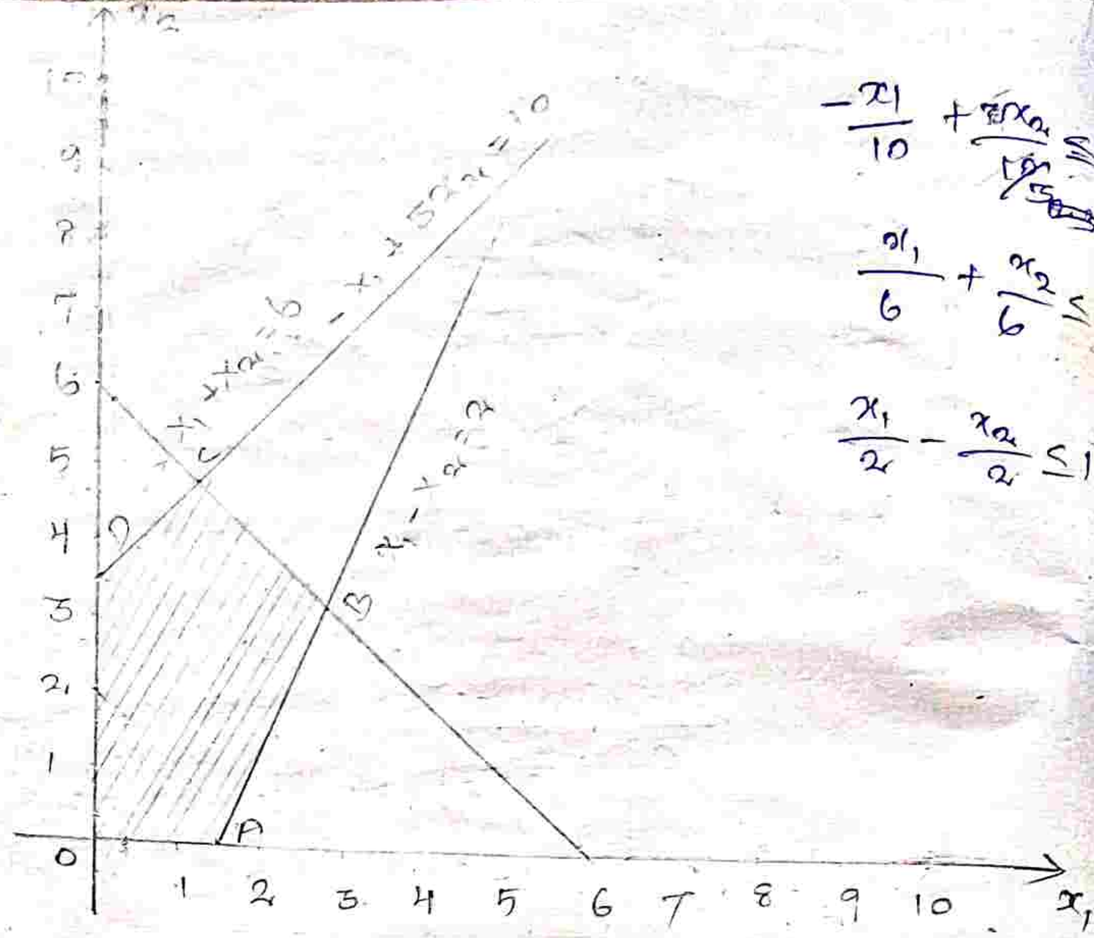
305. use the graphical method to solve the following L.P.P

$\min z = -x_1 + 2x_2.$

subject to the containts : $-x_1 + 3x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$ and



$$-\frac{x_1}{10}+\frac{\cancel{5}x_2}{\cancel{10/5}}\leq$$

$$\frac{x_1}{6}+\frac{x_2}{6}\leq 1$$

$$\frac{x_1}{2}-\frac{x_2}{2}\leq 1$$

Point	(x_1, x_2)	$z=-x_1+2x_2$
O	(0,0)	0
A	(2,0)	-2
B	(4,2)	0
C	(2,4)	6
D	(0,10/3)	20/3.

min $z = -2$

$x_1 = 2$

$x_2 = 0$

The minimum of z occurs at the extreme point A(2,0) optium solution of L.P.P is

$x_1 = 2$ $x_2 = 0$ minimum = -2.

30 (b). use the graphical method to solve the following L.P.P

max $z = 2x_1 + 3x_2$

subject to the constraints :-

~~$x_1 + 5x_2 \le 10$~~ $x_1 + x_2 \le 30$

~~$x_1 + x_2 \le 5$~~ $x_1 - x_2 \ge 0, x_2 \ge 3$

~~$x_1 - x_2 \le 2$~~ $0 \le x_1 \le 20$

$0 \le x_2 \le 12.$

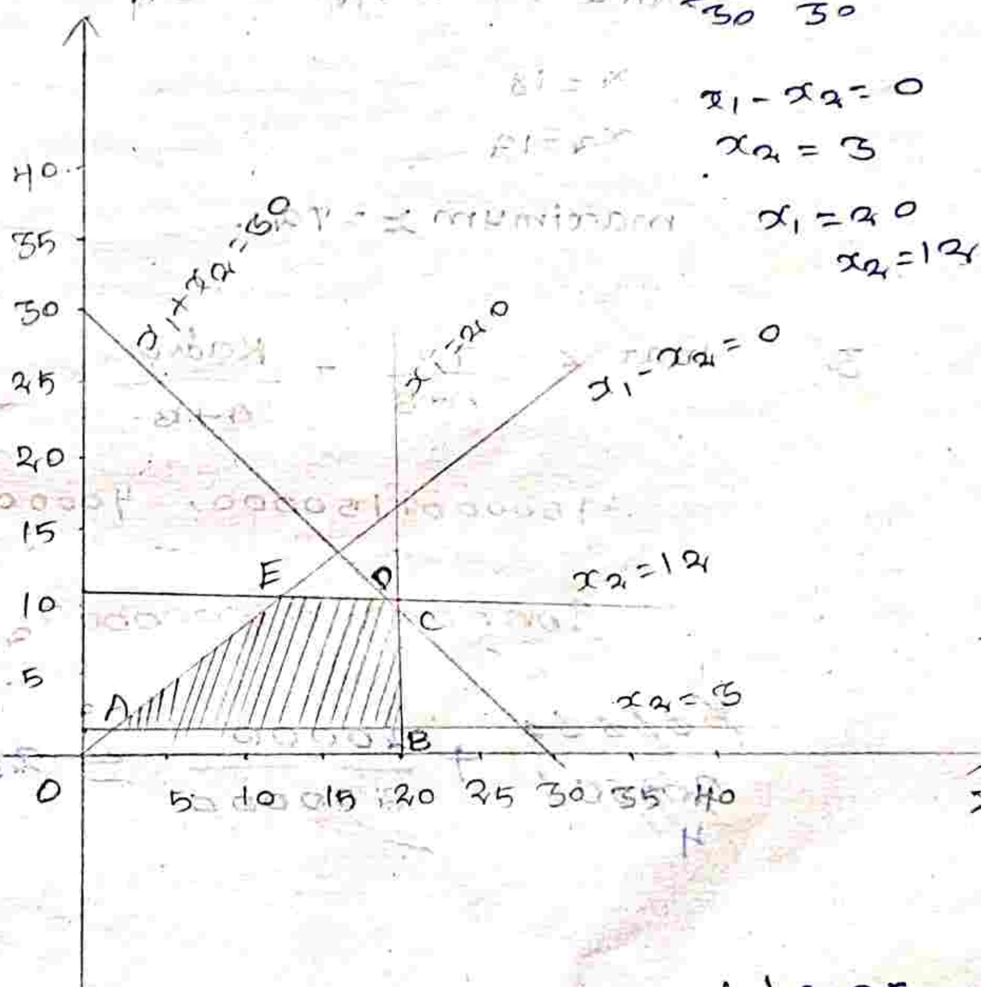
$\frac{x_1}{30} + \frac{x_2}{30} = 1$

$x_1 - x_2 = 0$

$x_2 = 3$

$x_1 = 20$

$x_2 = 12$



The condinates of the extreme points of the feasible region are.

A=(3,3) , D=(18,12)

B=(20,3) E=(12,12)

C = (20,10)

Extreme point	(x_1, x_2)	$Z = 2x_1 + 3x_2$
A	(5,5)	15
B	(20,5)	49
C	(20,10)	70
D	(18,12)	72
E	(12,12)	60

The maximum value of z occurs at the extreme point D (18, 12)

Hence, the optium solution is

$x_1 = 18$

$x_2 = 12$

maximum $z = 72$.

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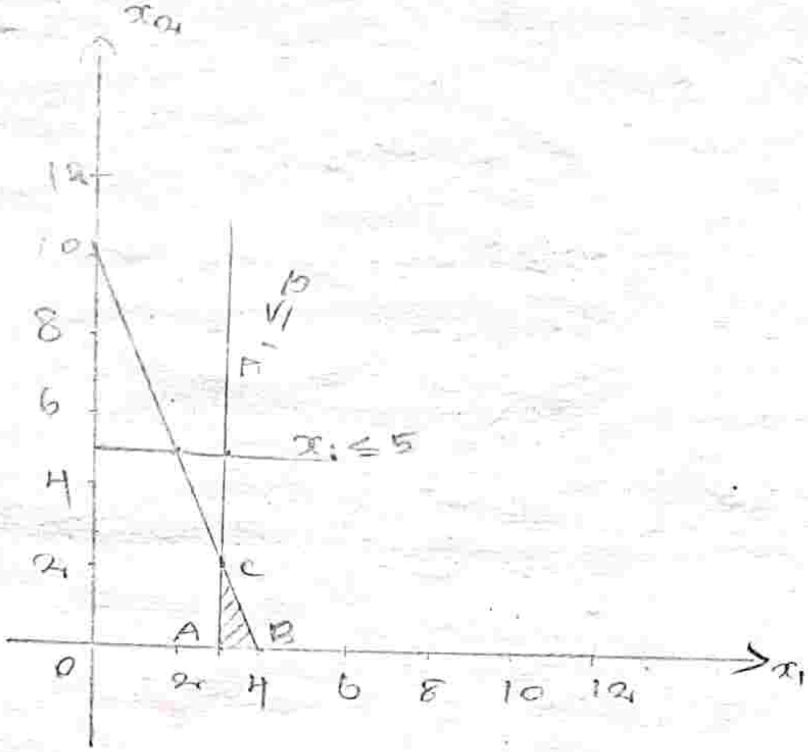
307. max $z = \frac{Tv}{A+B} + \frac{Radio}{A+B}$.

750000+150000, 40000 + 2,60000

$900000x_1 + 500000x_2$

$$\frac{\overset{1}{\cancel{500000}}}{\underset{4}{\cancel{2000000}}} + \frac{\overset{1}{\cancel{200000}}}{\underset{10}{\cancel{2000000}}} \leq 200000$$



point	(x_1, x_2)	Z
A	$(3,0)$	27,00,000
B	$(4,0)$	36,00,000
C	$(3, 5/2)$	34,50,000

$9(3) = 27$

$3(\frac{5}{2})$.

$= 15 + 27$

$= 30$

$2 \;\; 2$

max $z = 36,00,000$

$x_1 = 4, \; x_2 = 0.$

---

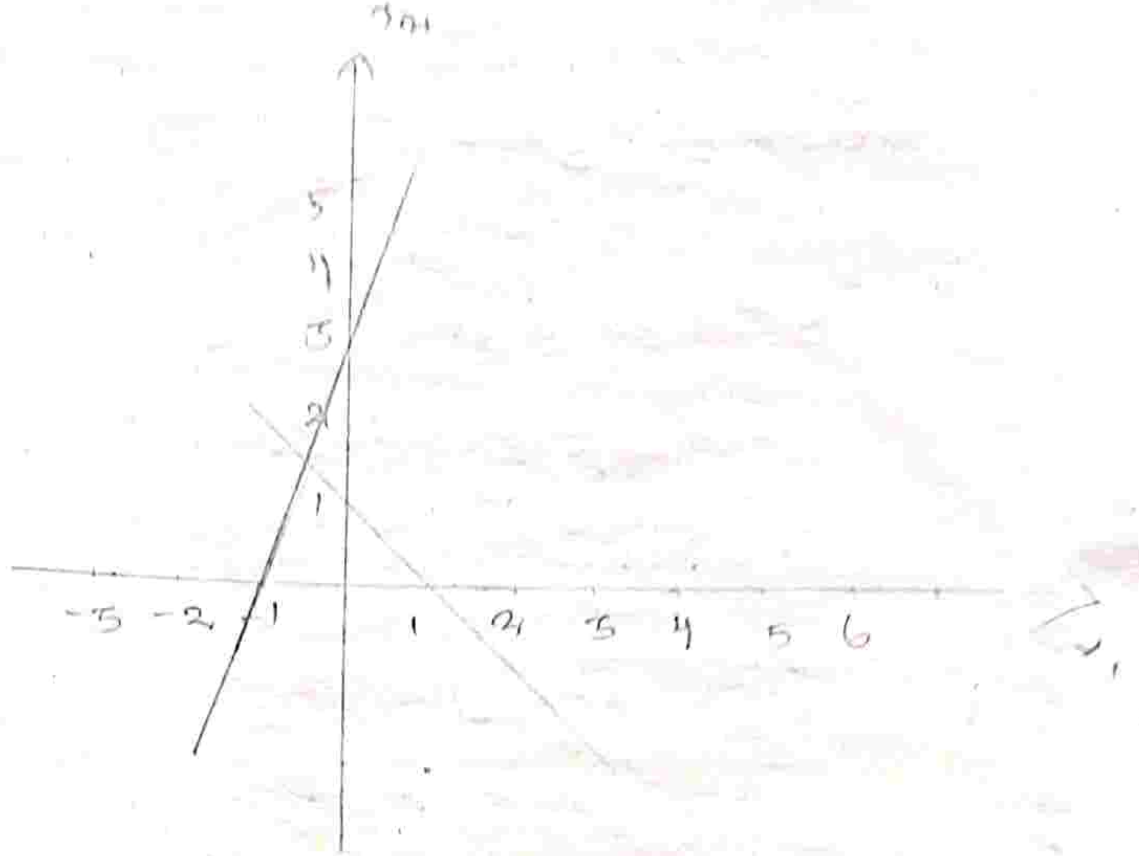
33 4. max $z = x_1 + x_2$

$x_1 + x_2 \leq 1$

$-3x_1 + x_2 \geq 3$.

$x_1, x_2 \geq 0$

$x_1 = -x_2$



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33 $\max z = 10x_1 + 6x_2$

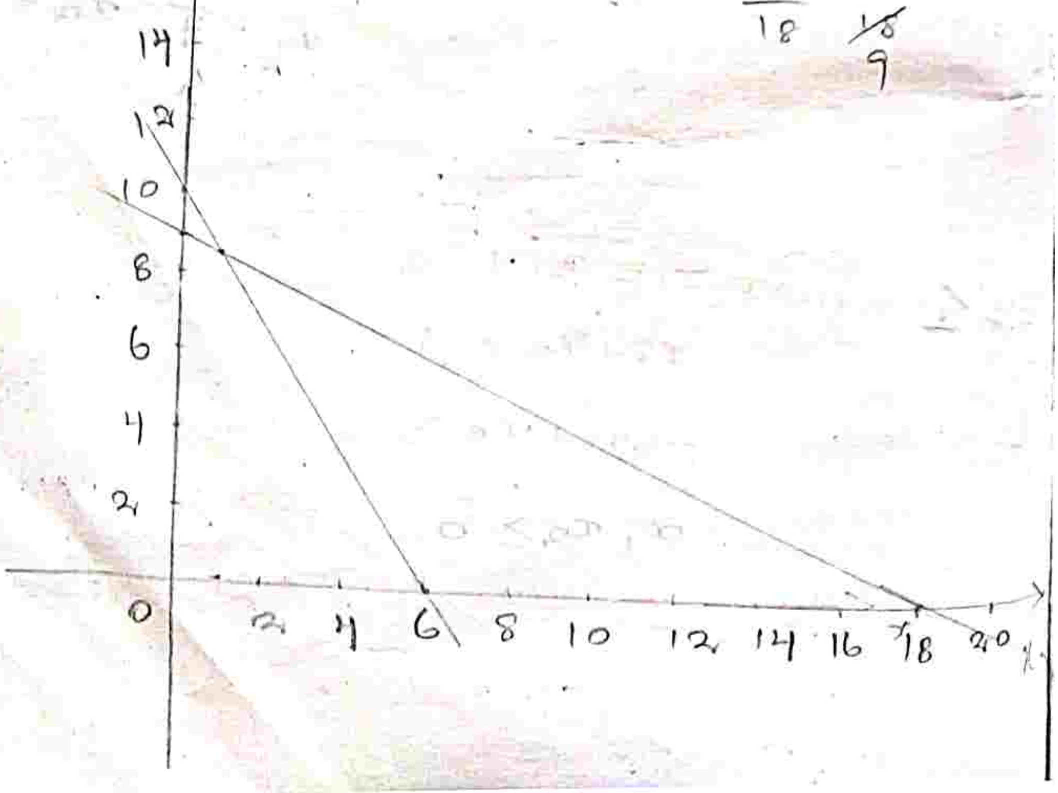
$5x_1 + 3x_2 \leq 30$

$x_1 + 2x_2 \leq 18$

$x_1, x_2 \geq 0$

$$\frac{\overset{1}{\cancel{5}}x_1}{\underset{6}{\cancel{30}}} + \frac{\overset{1}{\cancel{3}}x_2}{\underset{10}{\cancel{30}}}$$

$$\frac{x_1}{18} + \frac{\overset{1}{\cancel{2}}x_2}{\underset{9}{\cancel{18}}} =$$



Optimal solution method.

$\max Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Constraints $\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le = \ge b_1 \\ a_{m1}x_1 + a_{m2}x_2 + \cdots \le = \ge b_m \end{cases}$

$\sum a_{ij} x_j \le b_j$ slack variable (s_i)

$\sum a_{ij} x_j + s_i = b_j$

$\max z = cx$

$[Ax = b]$

surplus variable

$\sum a_{ij} x_j \ge b_i$

$\sum a_{ij} x_j - S_i = b_j$

$\max z = cx$

$Ax = b,\ x \ge 0$

Let x_1, x_2 be two feasible solution

$Ax_1 = b$

$Ax_2 = b,\ x_1, x_2 \ge 0$

$x = \lambda x_1 + (1-\lambda) x_2$

$Ax = A(\lambda x_1 + (1-\lambda) x_2)$

$= \lambda Ax_1 + (1-\lambda) Ax_2$

$= \lambda b + (1-\lambda) b = b$

3. min $z = 2x_1 + x_2 + 4x_3$ (standard Form)

subject to

$$-2x_1 + 4x_2 \le 4$$

$$x_1 + 2x_2 + x_3 \ge 5$$

$$2x_1 + 3x_3 \le 2$$

$x_1, x_2 \ge 0$ & x_3 unrestricted

$$x_3 = (x_3' - x_3'')$$

soln

$$\max(-z) = -2x_1 - x_2 - 4x_3$$

$$-2x_1 + 4x_2 + S_1 = 4 \longrightarrow (1)$$

$$x_1 + 2x_2 + (x_3' - x_3'') - S_3 = 5 \longrightarrow (3)$$

$$2x_1 + 3(x_3' - x_3'') + S_2 = 2 \longrightarrow (2)$$

---

2. basic solution

$x_1 + 2x_2 + x_3 = 4$

$2x_1 + x_2 + 5x_3 = 5$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = b$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$x_1 + 2x_2 = 4$

$4x_1 + \cancel{2}x_2 = 10$

$-3x_1 = -6$

$\boxed{x_1 = 2}$

$x_1 + 2x_2 = 4$

$(\times 2)\ 2x_1 + x_2 = 5$

$x_1 + \cancel{2}x_2 = 4$

$4x_1 + \cancel{2}x_2 = 10$

(-) (-) (-)

$-3x_1 = -6$

$\boxed{x_1 = 2}$

P.NO 100

(4))

$\max z = 4x_1 + 10x_2$

subject

$2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_1 + 3x_2 \leq 90$

$x_1, x_2 \geq 0$

solu:-

$\max z = 4x_1 + 10x_2 + 0s_1 + 0s_2 + 0.s_3$

$2x_1 + x_2 + s_1 = 50$

$2x_1 + 5x_2 + s_2 = 100$

$2x_1 + 3x_2 + s_3 = 90$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

C_j	C_j	4	10	0	0	0
C_B	y_i x_B	x_1	x_2	s_1	s_2	s_3
0	s_1 50	2	1	1	0	0
0	s_2 100	2	$\textcircled{5}$	0	1	0
0	s_3 90	2	3	0	0	1

x_B/x_2

$50/1 = 50$

$100/5 = 20$

$90/3 = 30$

min = 20

$Z_j = C_B(a_i) = 0\ 0\ 0\ 0\ 0$

$Z_j - C_j$ $-4\ -10\ 0\ 0\ 0$

most negative -10.

most negative lead = 5

2nd equ ÷ 5

100 2 5 0 1 0

÷5 20 $\frac{2}{5}$ 1 0 $\frac{1}{5}$ 0 $\rightarrow$ (2)

50 2 1 1 0 0

20 $\frac{2}{5}$ 1 0 $\frac{1}{5}$ 0

$2 \cdot \frac{2}{5}$

$\frac{10 \cdot 2}{5}$ 4

50 $\frac{8}{5}$ 0 1 $-\frac{1}{5}$ 0 $\rightarrow$ (1)

90 2 3 0 0 1

×3 60 $\frac{6}{5}$ 3 0 $\frac{3}{5}$ 0

30 $\frac{4}{5}$ 0 0 $-\frac{3}{5}$ 1 $\rightarrow$ (3)

		C_j	4	10	0	0	0
C_B	Y_i	x_B	x_1	x_2	S_1	S_2	S_3
0	S_1	30	$\frac{8}{5}$	0	1	$-\frac{1}{5}$	0
10	x_2	20	$\frac{2}{5}$	1	0	$\frac{1}{5}$	0
0	S_3	30	$\frac{4}{5}$	0	0	$-\frac{3}{5}$	1

$\frac{2}{10} \times 20$

$Z_j = C_B X_j$ 200 4 10 0 2 0

$Z_j - C_j$ 0 0 10 0 2 0

$Z_j - C_j \geq 0$

optium $Z = 200$

$x_2 = 20$ $x_1 = 0$, $4x_1 + 10x_2$

$4(0) + 10(20) = 0 + 200$

$= 200$

712. Use simplex method

minimize $z = x_2 - 3x_3 + 2x_5$ subject to the

constraints

$3x_2 - x_3 + 2x_5 \leq 7$

$-2x_2 + 4x_3 \leq 12$

$-4x_2 + 3x_3 + 8x_5 \leq 10$

$x_1 \geq 0, x_3 \geq 0, x_5 \geq 0$

Soln:-

max z

~~minimize~~ $z = -x_1 + 3x_2 + 2x_3 + 0.s_1 + 0.s_2 + 0s_3$

$3x_1 - x_2 + 2x_3 + s_1 = 7$

$-2x_1 + 4x_2 + s_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

	Cj		-1	3	-2	0	0	0	
CB	yi	xB	x_1	x_2	x_3	s_1	s_2	s_3	x_B/x_2
0	s_1	7	3	-1	2	1	0	0	$7/-1 = -7$
0	s_2	12	-2	$\boxed{4}$	0	0	1	0	$12/4 = 3$
0	s_3	10	-4	3	8	0	0	1	$10/3 = 3$

$z_j = C_B x_i$ = 0 0 0 0 0 0

1 -3 2 0 0 0

$z_j - c_j$ = most negative -3, min = 3

lead = 4.

12 -2 4 0 0 1 0

(÷4) 3 $-1/2$ 1 0 0 $1/4$ 0 → (2)

7 3 -1 2 1 0 0

(×1) 3 $-1/2$ 1 0 0 $1/4$ 0

10 $5/2$ 0 2 1 $1/4$ 0 → (1)

$3 - 1/2$

$\frac{6-1}{2}$

10 -4 3 8 0 0 1

(×3) 9 $-3/2$ 3 0 0 $3/4$ 0

(-) (+) (-) (-) (-) (-) (-)

1 $-5/2$ 0 8 0 $-3/4$ 1 → (3)

$10/(5/2)$ $20/5$

$-1/2$

$5/2$

		cj	-1	+3	-2	0	0	0
CB	yi	xB	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	10	$\boxed{5/2}$	0	2	1	$1/4$	0
3	x_2	3	$-1/2$	1	0	0	$1/4$	0
0	s_3	1	$-5/2$	0	8	0	$-3/4$	1

x_B/x_1

$\frac{10}{5/2} = 4$

$10 \times 2/5 = 4$

$3/(-1/2) = 3\times$

$= -6$

$1 \times -5/2 = 1\times$

$= -2/5$

$= -0.4$

Zj = CB xi = $-3/2$ 3 0 0 $3/4$ 0

Zj - cj = $-1/2$ 0 2 0 $3/4$ 0

most negative $-1/2$ min = 4

10 $\frac{5}{2}$ 0 2 1 $\frac{1}{4}$ 0

(÷5/2) 4 1 0 $\frac{4}{5}$ $\frac{2}{5}$ $\frac{1}{10}$ 0 →(1)

3 $-\frac{1}{2}$ 1 0 0 $\frac{1}{4}$ 0

(×1/2) 2 $\frac{1}{2}$ 0 $\frac{2}{5}$ $\frac{1}{5}$ $\frac{1}{20}$ 0

5 0 1 $\frac{2}{5}$ $\frac{1}{5}$ $\frac{3}{10}$ 0 →(2)

1 $-\frac{5}{2}$ 0 8 0 $-\frac{3}{4}$ 1

10 $\frac{5}{2}$ 0 2 1 $+\frac{1}{4}$ 0

11 0 0 10 1 $-\frac{1}{2}$ 1 →(3)

	Cj		-1	3	-2	0	0	0
C_B	y_i	x_B	x_1	x_2	x_3	S_1	S_2	S_3
-1	x_1	4	1	0	$\frac{4}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	0
3	x_2	5	0	1	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{3}{10}$	0
0	S_3	11	0	0	10	1	$-\frac{1}{2}$	1

$z_j = C_B x_j$ 11 -1 3 $\frac{2}{5}+2$ $\frac{1}{5}$ $\frac{4}{5}$ 0

$z_j - c_j$ 0 0 $\frac{12}{5}$ $\frac{1}{5}$ $\frac{4}{5}$ 0

$x_1 = 4$ $x_2 = 5$, $x_3 = 0$

min z = $-$max z_1 = -11

subject to equation $z^* = 4 - 15 \Rightarrow x_1 - 3x_2 + 2x_3$
$4 - 15 + 0$

$z^* = -11$

413 Find the maximum value of $z = 107x_1 + x_2 + 2x_3$

subject to constraints

$$14x_1 + x_2 - 6x_3 + 3x_4 = 7$$

$$16x_1 + x_2 - 6x_3 \leq 5$$

$$3x_1 - x_2 - x_3 \leq 0$$

$$x_1\ x_2\ x_3,\ x_4 \geq 0$$

Solu :-

1 ~~$107x_1$~~ - max $z = 107x_1 + x_2 + 2x_3 + 0s_1 + 0s_2 + 0s_3$

$$14x_1 + x_2 - 6x_3 + 3x_4 = 7$$

$$\frac{14}{3}x_1 + \frac{1}{3}x_2 - \overset{2}{\cancel{\frac{6}{3}}}x_3 + s_1 = \frac{7}{3}$$

$$16x_1 + x_2 - 6x_3 + s_2 = 5$$

$$3x_1 - x_2 - x_3 + s_3 = 0$$

	C_j		107	1	2	0	0	0	
C_B	y_i	x_B	x_1	x_2	x_3	s_1	s_2	s_3	x_B/x_1
0	s_1	$7/3$	$14/3$	$1/3$	-2	1	0	0	$\frac{7/3}{14/3} = \frac{7}{3}$ $= \frac{1}{2}$
0	s_2	5	16	1	-6	0	1	0	$5/16 = 3$
0	s_3	0	③	-1	-1	0	0	1	$0/3 = 0$

$z_j = C_B x_i \quad = 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

$z_j - c_j \quad = -107 \quad -1 \quad -2 \quad 0 \quad 0 \quad 0$

most negative -107 min $= 0$

0 3 -1 -1 0 0

(÷3) 0 1 $-\frac{1}{3}$ $-\frac{1}{3}$ 0 0 $\frac{1}{3}$ → (3)

$\frac{7}{3}$ $\frac{14}{3}$ $\frac{1}{3}$ -2 1 0 0

(×14/3) 0 $\frac{14}{3}$ $-\frac{14}{9}$ $-\frac{14}{9}$ 0 0 $\frac{14}{9}$

(-) (-) (+) (+) (-) (-) (-)

$\frac{7}{3}$ 0 $\frac{17}{9}$ $-\frac{4}{9}$ 1 0 $-\frac{14}{9}$ → (1)

$\frac{1}{3}+\frac{14}{9}$

$\frac{3+14}{9}$

$\frac{17}{9}$

$-2+\frac{14}{9}$

$\frac{-18+14}{9}$

$\frac{1}{3}\times\frac{x}{\alpha_3}+3$

$\frac{1}{9}$

5 16 1 -6 0 1 0

(×16) 0 16 $-\frac{16}{3}$ $-\frac{16}{3}$ 0 0 $\frac{16}{3}$

(-) (-) (+) (+) (-) (-) (-)

5 0 $\frac{19}{3}$ $-\frac{2}{3}$ 0 1 $-\frac{16}{3}$ → (2)

$1+\frac{16}{3}$

$\frac{3+16}{3}$

$=\frac{19}{3}$

$-\frac{102}{3}$

$-\frac{107}{3}$

$-\frac{107}{3}$

	cj		107	1	2	0	0	0
C_B	y_i	x_B	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	$\frac{7}{3}$	0	$\frac{17}{9}$	$-\frac{4}{9}$	1	0	$-\frac{14}{9}$
0	s_2	5	0	$\frac{19}{3}$	$-\frac{2}{3}$	0	1	$-\frac{16}{3}$
107	x_1	0	1	$-\frac{1}{3}$	$-\frac{1}{3}$	0	0	$\frac{1}{3}$

$z_j = C_B x_i$ = 0 107 $-\frac{107}{3}$ $-\frac{107}{3}$ 0 0 $\frac{107}{3}$

$z_j - c_j$ = 0 $-\frac{110}{3}$ $-\frac{113}{3}$ 0 0 $+\frac{107}{3}$

$\frac{107}{3}-1$

$\frac{-107-3}{3}$

$-\frac{110}{3}$

optium z = 0

$x_1 = 0,\ x_2 = 0,\ x_3 = 0,\quad z = 107(0) + (0) + 2(0) = 0$

417

use simplex method

maximize $z = 2x_1 - x_2 + x_3$ subject to constraint;

$3x_1 + x_2 + x_3 \leq 60$

$x_1 - x_2 + 2x_3 \leq 10$

$x_1 + x_2 - x_3 \leq 20$

$x_1 \; x_2 \; x_3 \geq 0$

Soln :-

max $z = 2x_1 - x_2 + x_3 + 0s_1 + 0s_2 + 0s_3$

$3x_1 + x_2 + x_3 + s_1 = 60$

$x_1 - x_2 + 2x_3 + s_2 = 10$

$x_1 + x_2 - x_3 + s_3 = 20$

	c_j		2	-1	1	0	0	0	$\frac{x_B}{x_1}$
C_B	y_i	x_B	x_1	x_2	x_3	s_1	s_2	s_3	
0	s_1	60	3	1	1	1	0	0	$60/3 = 20$
0	s_2	10	①	-1	2	0	1	0	$10/1 = 10$
0	s_3	20	1	1	-1	0	0	1	$20/1 = 20$

$z_j = C_B x_j$ = 0 0 0 0 0 0

$z_j - c_j$ = -2 1 -1 0 0 0

most negative -2, min = 10

10 1 -1 2 0 1 0

(÷1) 10 1 -1 2 0 1 0 → (2)

60 3 1 1 1 0 0

(×3) 30 3 -3 6 0 3 0

(-) (-) (+) (-) (-) (-) (-)

30 0 4 -5 1 -3 0 → (1)

20 1 1 -1 0 0 1

(×1) +10 +1 -1 +2 0 +1 0

(-) (-) (+) (-) (-) (-) (-)

10 0 2 -3 0 -1 1 → (3)

	Cj		2	-1	1	0	0	0	
CB	yi	x_B	x_1	x_2	x_3	S_1	S_2	S_3	x_B/x_2
0	S_1	30	0	4	-5	1	-3	0	30/4=7.5
2	x_1	10	1	-1	2	0	1	0	10/-1=-10
0	S_3	10	0	②	-3	0	-1	1	10/2=5

$z_j = CB\ x_i =$ 2 -2 4 0 2 0

$z_j - c_j =$ 0 -1 3 0 2 0

most negative -1 min = 5

10 0 2 -3 0 -1 1

(÷2) 5 0 1 -3/2 0 -1/2 1/2 →(1)

30 0 4 -5 1 -3 0

(x4) 20 0 4 -6 0 -2 2

(-) (-) (-) (+) (-) (-) (-)

10 0 0 1 1 -1 -2 →(1)

10 1 -1 2 0 1 0

5 0 1 -3/2 0 -1/2 1/2 (+)

15 1 0 1/2 0 1/2 1/2 →(2)

		C_j	2	-1	1	0	0	0
C_B	y_i	x_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	10	0	0	1	1	-1	-2
2	x_1	15	1	0	1/2	0	1/2	1/2
-1	x_2	5	0	1	-3/2	0	-1/2	1/2

$z_j - C_B x_j = 25$ 2 -1 5/2 0 3/2 1/2

$z_j - c_j$ = 0 0 3/2 0 3/2 1/2

30 - 5

= 25

optium $z = 25$

$x_1 = 15, \quad x_2 = 5 \quad x_3 = 0$

subject to equation

$$z^* = 2x_1 - x_2 + x_3$$

$$= 2(15) - 5 + 0$$

$$= 30 - 5$$

$$z^* = 25$$

---

418. use simplex method to maximize $z = 3x_1 + 2x_2 + 5x_3$ subject to the constraints.

$x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_3 \leq 420$

$x_1, x_2, x_3 \geq 0$.

Solu :-

$\max z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

$x_1 + 2x_2 + x_3 + s_1 = 430$

$3x_1 + 2x_3 + s_2 = 460$

$x_1 + 4x_3 + s_3 = 420$

	C_j		3	2	5	0	0	0	
C_B	y_i	x_B	x_1	x_2	x_3	s_1	s_2	s_3	x_B/x_3
0	s_1	450	1	2	1	1	0	0	$\frac{450}{1}=4$
0	s_2	460	3	0	2	0	1	0	$\overset{230}{\frac{460}{2}}=$
0	s_3	420	1	0	$\boxed{4}$	0	0	1	$\overset{105}{\frac{420}{4}}=1$

$z_j = C_B x_i = 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

$z_j - c_j = -3 \quad -2 \quad -5 \quad 0 \quad 0 \quad 0$

most negative = -5 min = 105

$420 \quad 1 \quad 0 \quad 4 \quad 0 \quad 0 \quad 1$

$(\div 4)$ $105 \quad \frac{1}{4} \quad 0 \quad 1 \quad 0 \quad 0 \quad \frac{1}{4} \rightarrow (3)$

$\overset{210}{\cancel{450}} - 105$
105
325

$450 \quad 1 \quad 2 \quad 1 \quad 1 \quad 0 \quad 0$

$\frac{1}{4} \times 2$
$\frac{1}{2}$

$(\times 2)$ $105 \quad \frac{1}{4} \quad 0 \quad 1 \quad 0 \quad 0 \quad \frac{1}{4}$

(−) (−) (−) (−) (−) (−) (−)

$325 \quad \frac{3}{4} \quad 2 \quad 0 \quad 1 \quad 0 \quad -\frac{1}{4} \rightarrow (1)$

$\frac{15}{2}$
$\overline{10}$

$-\frac{1}{4} \quad 2-0$

$\frac{1-1}{4} \quad \frac{5}{4}$

$460 \quad 3 \quad 0 \quad 2 \quad 0 \quad 1 \quad 0$

$3-\frac{1}{2}$
$\frac{6-1}{2}$
$\frac{5}{2}$

$(\times 2)$ $210 \quad \frac{1}{2} \quad 0 \quad 2 \quad 0 \quad 0 \quad \frac{1}{2}$

(−) (−) (−) (−) (−) (−) (−)

$250 \quad \frac{5}{2} \quad 0 \quad 0 \quad 0 \quad 1 \quad -\frac{1}{2} \rightarrow (2)$

P-9

1

	Cj		3	2	5	0	0	0	
C_B	yi	x_B	x_1	x_2	x_3	S_1	S_2	S_3	$\frac{x_B}{x_2}$
0	S_1	525	$3/4$	$\textcircled{2}$	0	1	0	$-1/4$	$\frac{525}{2}$
0	S_2	250	$5/2$	0	0	0	1	$-1/2$	$\frac{250}{0}=\infty$
5	x_3	105	$1/4$	0	1	0	0	$1/4$	$\frac{105}{0}=\infty$

200

$z_j = C_B x_i =$ $5/4$ 0 5 0 0 $5/4$

$z_j - c_j =$ $-7/4$ -2 0 0 0 $5/4$

$5/4 - 3$

$\frac{5-12}{4}$

$\frac{-7}{4}$

most negative -2

optium z = 525

$x_1 = 0$ $x_2 = 0$ $x_3 = 105$

subject to equation

$x_1 = 0$, $x_2 = 0$, $x_3 = 105$ 525

$$\begin{array}{r} \overset{2}{1}05 \\ \times 5 \\ \hline 535 \end{array}$$

$z^* = 3x_1 + 2x_2 + 5x_3$

$3(0) + 2(0) + 5(105)$

$$\begin{array}{r} \overset{2}{1}05 \\ 5 \\ \hline 525 \end{array}$$

$z^* = 525$

423. Show that the L.P.P

maximize $z = 4x_1 + x_2 + 3x_3 + 5x_4$ subject contraints

$$4x_1 - 6x_2 - 5x_3 - 4x_4 \geq -20$$

$$3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$$

$$8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$$

$$x_1\ x_2\ x_3\ x_4 \geq 0$$

Solu :-

maximize $z = 4x_1 + x_2 + 3x_3 + 5x_4 + 0s_1 + 0s_2 + 0s_3$

$$4x_1 - 6x_2 - 5x_3 - 4x_4 - \cancel{s_1} \geq -20$$

$$\left[-4x_1 + 6x_2 + 5x_3 + 4x_4 + s_1 = 20\right]$$

$$3x_1 - 2x_2 + 4x_3 + x_4 + s_2 = 10$$

$$8x_1 - 3x_2 + 3x_3 + 2x_4 + s_3 = 20$$

	C_j		4	1	3	5	0	0	0	
C_B	y_i	x_B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	x_B/x_4
0	s_1	+20	-4	+6	+5	$\boxed{+4}$	+1	0	0	$+20/+4 =$
0	s_2	10	3	-2	4	1	0	1	0	$10/1 =$
0	s_3	20	8	-3	3	2	0	0	1	$20/2 =$

$Z_j = C_B x_i$ 0 0 0 0 0 0 0

$Z_j - C_j$ -4 -1 -3 -5 0 0 0

most negative -5 , min = 5

$20 \quad -4 \quad 6 \quad 5 \quad 4 \quad +1 \quad 0 \quad 0$

$(\div 4)$

$5 \quad -1 \quad \frac{3}{2} \quad \frac{5}{4} \quad 1 \quad +\frac{1}{4} \quad 0 \quad 0 \rightarrow (1)$

$10 \quad 3 \quad -2 \quad 4 \quad 1 \quad 0 \quad 1 \quad 0$

$(\times 1)$ $5 \quad -1 \quad \frac{3}{2} \quad \frac{5}{4} \quad 1 \quad +\frac{1}{4} \quad 0 \quad 0$

$(-) \quad (+) \quad (-) \quad (-) \quad (-) \quad (+) \quad (-) \quad (-)$

$5 \quad 4 \quad -\frac{7}{2} \quad \frac{11}{4} \quad 0 \quad -\frac{1}{4} \quad 1 \quad 0 \rightarrow (2)$

$20 \quad 8 \quad -3 \quad 3 \quad 2 \quad 0 \quad 0 \quad 1$

$(\times 2)$ $10 \quad -2 \quad 3 \quad \frac{5}{2} \quad 2 \quad +\frac{1}{2} \quad 0 \quad 0$

$(-) \quad (+) \quad (-) \quad (+) \quad (-) \quad (+) \quad (-) \quad (-)$

$10 \quad 10 \quad -6 \quad \frac{1}{2} \quad 0 \quad -\frac{1}{2} \quad 0 \quad 1 \rightarrow (3)$

	C_j		4	1	3	5	0	0	0	
C_B	y_B	x_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	x_B/x_1
5	x_4	5	-1	$\frac{3}{2}$	$\frac{5}{4}$	1	$+\frac{1}{4}$	0	0	$5/-1 = -5$
0	S_2	5	4	$-\frac{7}{2}$	$\frac{11}{4}$	0	$-\frac{1}{4}$	1	0	$5/4$
0	S_3	10	$\boxed{10}$	-6	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	1	$10/10 = 1$

$Z_j = C_B x_i \quad 25 \quad -5 \quad \frac{15}{2} \quad \frac{25}{4} \quad 5 \quad \frac{5}{4} \quad 0 \quad 0$

$Z_j - C_j \quad -9 \quad \frac{15}{2} \quad \frac{15}{4} \quad 0 \quad \frac{5}{4} \quad 0 \quad 0$

most negative -9, min $= 1$

$10 \quad 10 \quad -6 \quad 1/2 \quad 0 \quad -1/2 \quad 0 \quad 1$

$(\div 10) \quad 1 \quad 1 \quad -3/5 \quad 1/20 \quad 0 \quad -1/20 \quad 0 \quad 1/10 \rightarrow (3)$

$5 \quad (-1) \quad 3/2 \quad 5/4 \quad 1 \quad 1/4 \quad 0 \quad 0$

$(\times 1) \quad 1 \quad +1 \quad -3/5 \quad 1/20 \quad 0 \quad -1/20 \quad 0 \quad 1/10$

$6 \quad 0 \quad 9/10 \quad 13/10 \quad 1 \quad -1/5 \quad 0 \quad 1/10 \rightarrow (1)$

$5 \quad 4 \quad -7/2 \quad 11/4 \quad 0 \quad -1/4 \quad 1 \quad 0$

$(\times 4) \quad 4 \quad 4 \quad -12/5 \quad 1/5 \quad 0 \quad -1/5 \quad 0 \quad 3/5$

$(-) \quad (-) \quad (+) \quad (-) \quad (+) \quad (+) \quad (-) \quad (-)$

$1 \quad 0 \quad -1/10 \quad 5/20 \quad 0 \quad -1/20 \quad 1 \quad -2/5 \rightarrow (2)$

	C_j		4	1	3	5	0	0	0
C_B	y_B	x_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3
5	x_4	6	0	$9/10$	$13/10$	1	$-1/5$	0	$1/10$
0	S_2	1	0	$-1/10$	$5/20$	0	$-1/20$	1	$-2/5$
4	x_1	1	1	$-3/5$	$1/20$	0	$-1/20$	0	$1/10$

$z_j = C_B x_i \quad 54 \quad 4 \quad 21/10 \quad 65/10 \quad 5 \quad 4/5 \quad 0 \quad 9/10$

$z_j - c_j \quad 0 \quad 11/10 \quad 37/10 \quad 0 \quad 4/5 \quad 0 \quad 9/10$

$\frac{1/2}{10}$

$\frac{1/3}{10}$

$\frac{3/5}{1/5}$

$\frac{1-20}{4}$

$\frac{3}{2} - \frac{3}{5}$

$\frac{15-6}{10}$

$\frac{9}{10}$

$\frac{1}{2} + \frac{12}{5}$

$\frac{-35+24}{10}$

$+5$

$\frac{5+20}{4}$

$\frac{25}{4}$

$\frac{11}{4} - 20$

$\frac{45}{10} - \frac{12}{5}$

$\frac{45-24}{10}$

$125+80$

$\frac{1}{4} = 1$

∴ optium $z^* = 34$

$x_1 = 1$ $x_2 = 0$ $x_3 = 0$ $x_4 = 6$

Subject to the constraint s

$z^* = 4x_1 + 1x_2 + 3x_3 + 5x_4$

$= 4(1) + 1(0) + 3(0) + 5(6)$

$= 4 + 30$

$z^* = 34$

---

1. Formulate the dual in LPP.

$\max z = 5x_1 + 3x_2$

$3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$ $x_1, x_2 \geq 0$

Solu:-

$\max z = 5x_1 + 3x_2 + 0s_1 + 0s_2$

$3x_1 + 5x_2 + s_1 = 15$

$5x_1 + 2x_2 + s_2 = 10$

$\left\{ \begin{array}{l} w_1 + 0w_2 \\ 0w_1 + w_2 \end{array} \right. \geq 0$

$\min z = 15w_1 + 10w_2$

$3w_1 + 5w_2 \geq 5$

$5w_1 + 2w_2 \geq 3$

Two phase Method.

425 use two phase ~~me~~ simplex method to maximize $z = 5x_1 - 4x_2 + 3x_3$ subject to the constraints

$$2x_1 + x_2 - 6x_3 = 20$$

$$6x_1 + 5x_2 + 10x_3 \leq 76$$

$$8x_1 - 3x_2 + 6x_3 \leq 50, \quad x_1, x_2, x_3 \geq 0$$

Solu :-

maximize. $z = 5x_1 - 4x_2 + 3x_3 + 0S_1 + 0S_2 - A_1$

$$2x_1 + x_2 - 6x_3 + A_1 = 20$$

$$6x_1 + 5x_2 + 10x_3 + S_1 = 76$$

$$8x_1 - 3x_2 + 6x_3 + S_2 = 50$$

phase - I

	c_j		0	0	0	0	0	-1	
C_B	y_B	x_B	x_1	x_2	x_3	S_1	S_2	A_1	x_B/x_1
-1	A_1	20	2	1	-6	0	0	1	$20/2 = 10$
0	S_1	76	6	5	10	1	0	0	$76/6 = 12.7$
0	S_2	50	$\boxed{8}$	-3	6	0	1	0	$50/8 = 6.25$

$z_j = C_B x_i$ = -2 -1 6 0 0 -1

$z_j - c_j$ = $\boxed{-2}$ -1 6 0 0 0

negative -2 , min = 6.25

50 8 -3 6 0 1 0

(÷8) $\frac{25}{4}$ 1 $-\frac{3}{8}$ $\frac{3}{4}$ 0 $\frac{1}{8}$ 0 → (5)

20 2 1 -6 0 0 1

(×2) $\frac{25}{2}$ 2 $-\frac{3}{4}$ $\frac{3}{2}$ 0 $\frac{1}{4}$ 0

(-) (-) (+) (-) (-) (-) (-)

$\frac{15}{2}$ 0 $\frac{7}{4}$ $-\frac{15}{2}$ 0 $-\frac{1}{4}$ 1 → (1)

76 6 5 10 1 0 0

(×6) (1) $\frac{75}{2}$ 6 $-\frac{9}{4}$ $\frac{9}{2}$ 0 $\frac{3}{4}$ 0

(-) (-) (+) (-) (-) (-) (-)

$\frac{77}{2}$ 0 $+\frac{29}{4}$ $\frac{11}{2}$ 1 $-\frac{3}{4}$ 0 → (2)

	cj		0	0	0	0	0	-1	
CB	yB	xB	x_1	x_2	x_3	S_1	S_2	A_1	x_B/x_2
-1	A_1	$\frac{15}{2}$	0	$\boxed{\frac{7}{4}}$	$-\frac{15}{2}$	0	$-\frac{1}{4}$	1	$\frac{15}{2}/\frac{7}{4} = \frac{15}{2}\times$ $\boxed{=\frac{30}{7}}$
0	S_1	$\frac{77}{2}$	0	$\frac{29}{4}$	$\frac{11}{2}$	1	$-\frac{3}{4}$	0	$\frac{77}{2}/\frac{29}{4} = \frac{77}{2}\times\frac{4}{2}$ $\boxed{=\frac{77}{14.5}}$
0	x_1	$\frac{25}{4}$	1	$-\frac{3}{8}$	$\frac{3}{4}$	0	$\frac{1}{8}$	0	$\frac{25}{4}/-\frac{3}{8} = \frac{25}{4}\times\frac{8}{-3}$ $\boxed{=-\frac{50}{3}}$

$z_j = C_B x_i$ = 0 $-\frac{7}{4}$ $+\frac{15}{2}$ 0 $\frac{1}{4}$ 1

$z_j - c_j$ = 0 $-\frac{7}{4}$ $\frac{15}{2}$ 0 $\frac{1}{4}$ 0

$\boxed{\text{most negative} = -\frac{7}{4}}$ $\boxed{\text{min} = \frac{30}{7}}$

$\frac{15}{2}$ 0 $\frac{7}{4}$ $-\frac{15}{2}$ 0 $-\frac{1}{4}$ 1

$(\div \frac{7}{4})$ $\frac{30}{7}$ 0 1 $-\frac{30}{7}$ 0 $-\frac{1}{7}$ $\frac{4}{7}$ $\rightarrow (1)$

$\frac{30}{7} \times \frac{29}{4}$

$\frac{120 \times 203}{28}$

$\frac{77}{2}$ 0 $\frac{29}{4}$ $\frac{11}{2}$ 1 $-\frac{5}{4}$ 0

$(\times \frac{29}{4})$ $\frac{435}{14}$ 0 $\frac{29}{4}$ $-\frac{435}{14}$ 0 $-\frac{29}{28}$ $\frac{29}{7}$

(−) (−) (−) (+) (−) (+) (−)

$\frac{77}{2} - \frac{435}{14}$

$\frac{52}{7}$ 0 0 $\frac{256}{7}$ 1 $\frac{2}{7}$ $-\frac{29}{7}$ $\rightarrow (2)$

$-\frac{1}{7} \times \frac{29}{4}$

$-\frac{29}{28}$

$-\frac{435}{14}$

$\frac{435 - 77}{14}$

$\frac{358}{14}$

$\frac{4}{7} \times \frac{29}{4}$

$\frac{25}{4}$ 1 $-\frac{5}{8}$ $\frac{5}{4}$ 0 $\frac{1}{8}$ 0

$\frac{1}{8} + \frac{9}{14}$

$(\times \frac{5}{8})$ $\frac{45}{28}$ 0 $\frac{5}{8}$ $-\frac{45}{28}$ 0 $\frac{3}{56}$ $\frac{5}{14}$

$\frac{55}{7}$ 1 0 $-\frac{6}{7}$ 0 $\frac{1}{14}$ $\frac{5}{14}$ $\rightarrow (3)$

	c_j		0	0	0	0	0	−1
C_B	y_B	x_B	x_1	x_2	x_3	S_1	S_2	A_1
0	x_2	$\frac{30}{7}$	0	1	$-\frac{30}{7}$	0	$-\frac{1}{7}$	$\frac{4}{7}$
0	S_1	$\frac{52}{7}$	0	0	$\frac{256}{7}$	1	$\frac{2}{7}$	$-\frac{29}{7}$
0	x_1	$\frac{55}{7}$	1	0	$-\frac{6}{7}$	0	$\frac{1}{14}$	$\frac{5}{14}$

$z_j = C_B x_i$ = 0 0 0 0 0 0

$z_j - c_j$ = 0 0 0 0 0 1

Phase 2:-

$Z = 5x_1 - 4x_2 + 5x_3 + 0s_1 + 0s_2$

	cj		5	-4	5	0	0
C_B	y_B	x_B	x_1	x_2	x_3	s_1	s_2
-4	x_2	$50/7$	0	1	$-30/7$	0	$-1/7$
0	s_1	$52/7$	0	0	$256/7$	1	$2/7$
5	x_1	$55/7$	1	0	$-6/7$	0	$1/14$

$z_j = c_B x_i = 155$ 5 -4 $90/7$ 0 $13/14$

$z_j - c_j =$ 0 0 $69/7$ 0 $13/14$

optium Z=155

(3)<-

$x_1 = 55/7, \quad x_2 = 50/7, \quad x_3 = 0$

$Z = 5x_1 - 4x_2 + 5x_3$

$5\left(\frac{55}{7}\right) - 4\left(\frac{50}{7}\right) + 5(0) = \frac{275-120}{7} + 0 = \frac{155}{7}$

$Z = \frac{155}{7}$

24. Use two-phase simplex method to maximize $Z = 5x_1 + 5x_2$ subject to the constraints:

$2x_1 + x_2 \leq 1, \quad x_1 + 4x_2 \geq 6$ and $x_1, x_2 \geq 0$

-5/14

4/7 + 5/14

(20-30)/7

5 90/7

21

Solu:-

$Z = 5x_1 + 5x_2 + S - S_2 - A_1$

$2x_1 + x_2 + S_1 = 1$

$x_1 + 4x_2 + S_2 + A_1 = 6$

Phase - I

	cj		0	0	~~0~~	0	0	-1	x_B/x_2
CB	yB	xB	x_1	x_2	~~x_3~~	S_1	S_2	A_1	
0	S_1	7	2	$\circled{1}$	~~0~~	1	0	0	$\cancel{1}=1$
-1	A_1	6	1	4		0	-1	1	$6/4 = 3/2$

$zj = CB\,x_j$ ~~4~~ -1   -4   0   1   -1

$zj - cj$ -1   -4   0   1   0

most negative -4   min = 1

1   2   1   1   0   0 $\rightarrow$ (1)

6   1   4   0   -1   1

(×4)   (-)4   (-)8   (-)4   (-)4   (-)0   (-)0

2   -7   0   -4   -1   1 $\rightarrow$ (2)

	C_j		0	0	0	0	-1
C_B	y_B	x_B	x_1	x_2	S_1	S_2	A_1
0	x_2	1	2	1	1	0	0
-1	A_1	2	-7	0	-4	-1	1

$z_j = C_B x_i$ -2 7 0 4 1 -1

$z_j - c_j =$ 7 0 4 1 0

does not any feasible solution

---

4 29 use two-phase simplex method to

a) maximize $z = 10x_1 + 20x_2$ subject to constraints

$$2x_1 + x_2 = 1$$
$$x_1 + 2x_2 = 5 \quad x_1 \geq 0, x_2 \geq 0$$

Solu:-

maximize $z = 10x_1 + 20x_2$ ~~$+0S_1$~~ $- A_1 - A_2$

$$2x_1 + x_2 + A_1 = 1$$
$$x_1 + 2x_2 + A_2 = 5$$

phase 1

	c_j		0	0	-1	-1
c_B	y_B	x_B	x_1	x_2	A_1	A_2
-1	A_1	1	2	①	1	0
-1	A_2	5	1	2	0	1

x_B / x_2

$1/1 = 1$

$5/2 = 2.5$

$\boxed{\min = 1}$

$z_j = c_B x_j$ 4   -3   -3   -1   -1

$z_j - c_j$ ~~3~~ -3   -3   0   0   -1 -1

-1-5
,4

$\boxed{\text{most negative } -3}$

1   2   1   1   0 $\rightarrow$ (1)

5   1   2   0   1

(×2)   2   4   2   2   0

(−)   (−)   (−)   (−)   (−)

3   -3   0   -2   1 $\rightarrow$ (2)

	cj		0	0	-1	-1
CB	yB	xB	x_1	x_2	A_1	A_2
0	x_2	1	2	1	1	0
-1	A_2	3	-3	0	-2	1

$z_j = C_B x_i = \quad -3 \quad 3 \quad 0 \quad 2 \quad -1$

$z_j - c_j \quad 3 \quad 0 \quad 3 \quad 0$

Does not any feasible solution

Phase ~~II~~

b) minimize $z = 2x_1 + 4x_2$ subject to the constraints

~~CB yB xB x1 x2 A1~~

$2x_1 + x_2 \geq 14 , \quad x_1 + 3x_2 \geq 18 , \quad x_1 + x_2 \geq 1$

$x_1, \; x_2 \geq 0$

solu :-

maximize $z = -2x_1 - 4x_2 + S_1 + S_2 + S_3 + A_1 + A_2 +$

$2x_1 + x_2 - S_1 + A_1 = 14$

$x_1 + 3x_2 - S_2 + A_2 = 18$

$x_1 + x_2 - S_3 + A_3 = 12$

	c_j		0	0	0	0	0	1	1	1
C_B	y_B	x_B	x_1	x_2	s_1	s_2	s_3	A_1	A_2	A_3
1	A_1	14	2	1	-1	0	0	1	0	0
1	A_2	18	1	3	0	-1	0	0	1	0
1	A_3	12	1	1	0	0	-1	0	0	1

$z_j = C_B x_i$ 4   5   -1   -1   -1   1   1   1

$z_j - c_j$ 4   5   -1   -1   -1   0   0   0

does not possess any Feasible solution.

BIG-M METHOD (method of penalties)

426. Use penalty (or Big M) method to maximize $z = 6x_1 + 4x_2$ subject to the constraints.

$2x_1 + 3x_2 \leq 30$; $3x_1 + 2x_2 \leq 24$, $x_1 + x_2 \geq 3$

$x_1 \geq 0$ and $x_2 \geq 0$.

Solu :-

$\max z = 6x_1 + 4x_2 + 0s_1 + 0s_2 + 0s_3 - M_1$

$2x_1 + 3x_2 + s_1 = 30$

$3x_1 + 2x_2 + s_2 = 24$

$x_1 + x_2 - s_3 + A_1 = 3$.

	C_j		6	4	0	0	0	$-M_1$	
C_B	y_B	x_B	x_1	x_2	S_1	S_2	S_3	A_1	x_B/x
0	S_1	30	2	3	1	0	0	0	$30/2 =$
0	S_2	24	3	2	0	1	0	0	$24/3 =$
$-M_1$	A_1	3	①	1	0	0	-1	1	$3/1 =$

$Z_j = C_B x_i$ = $\overset{-3M_1}{\cancel{-M_1}}$, $-M_1$, $-M_1$, 0, 0, M_1, $-M_1$,

$Z_j - C_j$ = $-M_1-6$, $-M_1-4$, 0, 0, M_1, 0

most negative $-M_1-6$, min=3

3 1 1 0 0 -1 1 → (3)

30 2 3 1 0 0 0

(*2) 6 2 2 0 0 -2 2

(-) (-) (-) (-) (-) (+) (-)

24 0 1 1 0 2 -2 → (1)

24 3 2 0 1 0 0

(×3) 9 3 3 0 0 -3 3

(-) (-) (-) (-) (-) (+) (-)

15 0 -1 0 1 3 -3 → (2)

	C_j		6	4	0	0	0	
C_B	y_B	x_B	x_1	x_2	s_1	s_2	s_3	x_B/s_3
0	s_1	24	0	1	1	0	2	$24/2 = 12$
0	s_2	15	0	-1	0	1	$\textcircled{3}$	$15/3 = 5$
6	x_1	3	1	1	0	0	-1	$3/-1 = -3$

$z_j = C_B x_i$ 18 6 6 0 0 -6

$z_j - c_j$ 0 2 0 0 -6

most negative -6 min = 5

15 0 -1 0 1 3 $\cancel{3}$

÷5 5 0 $-\frac{1}{3}$ 0 $\frac{1}{3}$ 1 → (2)

24 0 1 1 0 2

(×2) 10 0 $-\frac{2}{3}$ 0 $\frac{2}{3}$ 2

(-) (-) (+) (-) (-) (-)

14 0 $\frac{5}{3}$ 1 $-\frac{2}{3}$ 0 → (1)

$1 + \frac{2}{3}$

$\frac{3+2}{3}$

$\frac{5}{3}$

3 1 1 0 0 -1

(×1) 5 0 $-\frac{1}{3}$ 0 $\frac{1}{3}$ 1

8 1 $\frac{2}{3}$ 0 $\frac{1}{3}$ 0 → (3)

$1 - \frac{1}{3}$

$\frac{3-1}{3}$

$\frac{2}{3}$

	c_j		6	4	0	0	0
C_B	y_B	x_B	x_1	x_2	s_1	s_2	s_3
0	S_1	14	0	$5/3$	1	$-2/3$	0
0	S_3	5	0	$-1/3$	0	$1/3$	1
6	x_1	8	1	$2/3$	0	$1/3$	0

$6/3 =$

$z_j = C_B x_j$ 48 6 4/~~3~~ 0 2/~~3~~ 0

$3/1$

$z_j - c_j$ 0 0 0 2 0

$4/1$

$z^* = 48$

optium $z = 48$

$x_1 = 8, x_2 = 0$, ~~$x_3 = 0$~~

subject to the constraints

$z^* = 6x_1 + 4x_2$

$= 6(8) + 4(0)$

$z^* = 48$

427 maximize $z = 3x_1 + 2x_2$ subject to the constraints

$2x_1 + x_2 \leq 2, \quad 3x_1 + 4x_2 \geq 12, \quad x_1, x_2 \geq 0$

Solu:-

$\max z = 3x_1 + 2x_2 + 0S_1 + 0S_2 - M_1$

$2x_1 + x_2 + S_1 = 2$

$3x_1 + 4x_2 - S_2 + A_1 = 12$

	cj		3	2	0	0	$-M_1$	
CB	y_B	x_B	x_1	x_2	S_1	S_2	A_1	x_B/x_2
0	S_1	2	~~2~~	$\boxed{1}$	1	0	0	$2/1 = \boxed{2}$
$-M_1$	A_1	12	3	4	0	-1	1	$\cancel{12/4} = 3$

$z_j = C_B x_i = -12M_1 \quad -3M_1 \quad -4M_1 \quad 0 \quad +M_1 \quad -M_1$

$z_j - c_j \quad -3M_1-3 \quad -4M_1-2 \quad 0 \quad M_1 \quad 0$ ~~$-M_1$~~

$-M_1 - M$

$2M_1$

(most negative $= -4M_1 - 2$), (min = 2)

2 2 1 1 0 0 →(1)

12 3 4 0 −1 1

$\frac{12}{4}$ 48

(×4) 8 8 4 4 0 0

(−) (−) (−) (−) (−) (−)

4 −5 0 −4 −1 1 →(2)

	c_j		3	2	0	0	$-M_1$
C_B	y_B	x_B	x_1	x_2	S_1	S_2	A_1
2	x_2	2	2	1	ϕ	0	0
$-M_1$	A_1	4	-5	0	-4	-1	1

$z_j = C_B x_i = -4M_1+4 \quad 5M_1+4 \quad 2 \quad 4M_1+2 \quad M_1 \quad -M_1$

$4-4M_1$ $\quad z_j - c_j \qquad 5M_1+1 \quad 0 \quad 4M_1+2 \quad M_1 \quad 0$

tibisiyal value (-) [illegible]

does not possess any feasible solution.

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428. Use penalty (or Big M) method to maximize $z = x_1+2x_2+3x_3-x_4$ subject to the constraints.

$x_1+2x_2+3x_3=15$

$2x_1+x_2+5x_3=20$

$x_1+2x_2+x_3+x_4=10$

$x_1\ x_2\ x_3\ x_4 \geq 0.$

Solu :-

$\max z = x_1+2x_2+3x_3-x_4-A_1-A_2$

$x_1+2x_2+3x_3+A_1=15$

$2x_1+x_2+5x_3+A_2=20$

$x_1+2x_2+x_3+x_4 \cancel{+A_3} =10$

	c_j		1	2	3	-1	$-M_1$	$-M_2$	
C_B	y_B	x_B	x_1	x_2	x_3	x_4	A_1	A_2	x_B/x_3
$-M_1$	A_1	15	1	2	3	0	1	0	$15/3=5$
$-M_2$	A_2	20	2	1	$\textcircled{5}$	0	0	1	$20/5=4$
-1	x_4	10	1	2	1	1	0	0	$10/1=1$

$-M_1-20M_2-10$

$-3M$

$-15-20-10$

$-35-10$

-45

$-M_1$

$Z_j = C_B x_i$ $\quad -35M_1-10 \quad -3M-1 \quad -3M-2 \quad -8M-1 \quad -1 \quad -M_1 \quad -M_2$

$Z_j - C_j$ $\quad -3M-2 \quad -3M-4 \quad -8M-4 \quad 0 \quad 0 \quad 0$

most negative $= -8M-4$, min $= 4$

$20 \quad 2 \quad 1 \quad 5 \quad 0 \quad 0 \quad 1$

$(\div 5)$ $\quad 4 \quad 2/5 \quad 1/5 \quad 1 \quad 0 \quad 0 \quad 1/5 \rightarrow (2)$

$-1/5$

$2-3/$

$10-$

$7/$

$15 \quad 1 \quad 2 \quad 3 \quad 0 \quad 1 \quad 0$

$1-6/5$

$(\times 3)$ $\quad 12 \quad 6/5 \quad 3/5 \quad 3 \quad 0 \quad 0 \quad 3/5$

$\frac{5-6}{5}$

$(-) \quad (-) \quad (-) \quad (-) \quad (-) \quad (-) \quad (-)$

$3 \quad -1/5 \quad +7/5 \quad 0 \quad 0 \quad 1 \quad -3/5 \rightarrow (1)$

$-1/5$

$1-2/5$

$10 \quad 1 \quad 2 \quad 1 \quad 0 \quad 0$

$5-2$

$(\times 1)$ $\quad 4 \quad 2/5 \quad 1/5 \quad 1 \quad 0 \quad 0 \quad 1/5$

$2-1/5$

$(-) \quad (-) \quad (-) \quad (-) \quad (-) \quad (-) \quad (-)$

$\frac{10-1}{5}$

$(3) \leftarrow$ $\quad -6 \quad +3/5 \quad 9/5 \quad 0 \quad 1 \quad 0 \quad -1/5 \rightarrow (3)$

	Cj		1	2	+3	-1	$-M_1$	
C_B	y_B	x_B	x_1	x_2	x_3	x_4	A_1	x_B/x_2
$-M_1$	A_1	3	$-1/5$	$\boxed{7/5}$	0	0	1	$\frac{3}{7/5} = 3\times\frac{5}{7} = \frac{15}{7}$
3	x_3	4	$2/5$	$1/5$	1	0	0	$\frac{4}{1/5} = 4\times5 = 20$
-1	x_4	6	$-3/5$	$9/5$	0	1	0	$\frac{6}{9/5} = 6\times\frac{5}{9} = \frac{30}{9}$

$z_j = C_B x_i \quad -3M+6 \quad \frac{M}{5}+\frac{3}{5} \quad -\frac{7M}{5}-\frac{6}{5} \quad 3 \quad -1 \quad -M_1$

$z_j - c_j \quad \frac{M}{5}-\frac{2}{5} \quad -\frac{7M}{5}-\frac{16}{5} \quad 0 \quad 0 \quad 0$

most negative $-\frac{7M}{5}-\frac{16}{5}$, min = $\frac{15}{7}$

$3 \quad -\frac{1}{5} \quad \frac{7}{5} \quad 0 \quad 0 \quad 1$

$\left(\div \frac{7}{5}\right) \quad \frac{15}{7} \quad -\frac{1}{7} \quad 1 \quad 0 \quad 0 \quad \frac{5}{7} \rightarrow (1)$

$4 \quad \frac{2}{5} \quad \frac{1}{5} \quad 1 \quad 0 \quad 0$

$\left(\times \frac{1}{5}\right) \quad \frac{3}{7} \quad -\frac{1}{35} \quad \frac{1}{5} \quad 0 \quad 0 \quad \cancel{\frac{5}{35}} \ \frac{1}{7}$

$(-) \quad (+) \quad (-) \quad (-) \quad (-) \quad (-)$

$\frac{25}{7} \quad \frac{3}{35} \quad 0 \quad 1 \quad 0 \quad -\frac{1}{7} \rightarrow (2)$

$6 \quad -\frac{3}{5} \quad \frac{9}{5} \quad 0 \quad 1 \quad 0$

$\left(\times \frac{9}{5}\right) \quad \frac{27}{7} \quad -\frac{9}{35} \quad \frac{9}{5} \quad 0 \quad 0 \quad \frac{9}{7}$

$(-) \quad (+) \quad (-) \quad (-) \quad (-) \quad (-)$

$\frac{15}{7} \quad \frac{6}{7} \quad 0 \quad 0 \quad 1 \quad -\frac{9}{7} \rightarrow (3)$

$\frac{6}{5} - \frac{3}{5}$

$3 = \frac{3}{5}$

$+\frac{M}{5}$

$\frac{3}{5} - \frac{9}{5} - 6$

$\frac{6}{5} - 2 \quad \frac{6-10}{5}$

$-3M, -3$

$\frac{M}{5} + \frac{3}{5} - 1$

$\frac{15}{7} \times \frac{1}{5}$

$\frac{3}{5} + \frac{9}{35}$

$4 - \frac{3}{7} \quad \frac{28-3}{7}$

$\frac{2}{5} + \frac{1}{35}$

$\frac{15}{35}$

$\frac{5}{7} \times \frac{9}{5}$

$6 - \frac{27}{7}$

$\frac{42-27}{7}$

	c_j		1	2	3	-1	
c_B	y_B	x_B	x_1	x_2	x_3	x_4	x_B/x_1
2	x_2	$15/7$	$-1/7$	1	0	0	$\frac{15/7}{-1/7} = \frac{15}{7}\times\frac{7}{-1}$ $= -15$
3	x_3	$25/7$	$3/7$	0	1	0	$\boxed{8.3}$ $\frac{25}{7}\times\frac{7}{3} = \frac{25}{3}$
-1	x_4	$15/7$	$\boxed{6/7}$	0	0	1	$\boxed{2.5}$ $\frac{15}{7}\times\frac{7}{6} = \frac{15}{6}$ $\boxed{=2.5}$

$z_j = c_B x_i$ $\quad 90/7 \quad 1/7 \quad 2 \quad 3 \quad -1$

$z_j - c_j$ $\quad -6/7 \quad 0 \quad 0 \quad 0$

$\boxed{\text{most negative } -6/7}$, $\boxed{\min = 2.5}$

$15/7 \quad 6/7 \quad 0 \quad 0 \quad 1$ $\qquad \frac{49}{42}$ $\qquad \frac{15}{7}\times\frac{7}{6}$ $\qquad \frac{15}{42}$ $_{21}$

$(\div 6/7)$ $\quad \frac{\cancel{15}^{5}}{\cancel{42}_{2}} \quad 1 \quad 0 \quad 0 \quad 7/6 \rightarrow (3)$ $\qquad \frac{15}{6}\times\frac{1}{7}$ $\qquad \frac{15}{6}\times\frac{1}{7}$ $\quad \frac{5}{35}$

$15/7 \quad -1/7 \quad 1 \quad 0 \quad 0$ $\qquad -1/7$ $\quad \frac{15}{\frac{35}{7}}$

$(\times 1/7)$ $\quad \frac{\cancel{15}^{5}}{14} \quad 1/7 \quad 0 \quad 0 \quad 1/6$ $\qquad \frac{7}{6}\times\frac{1}{7}$

$\frac{15}{7}+\frac{15\times 7}{42\times 6}$

$5/2 \quad 0 \quad 1 \quad 0 \quad 1/6 \rightarrow (1)$ $\qquad \frac{15}{7}+\frac{3}{7}$ $\quad \frac{10+15}{7}$ $\quad \frac{18}{7}$ $\quad \frac{17}{15}$ $\frac{}{7}$

$25/7 \quad 3/7 \quad 0 \quad 1 \quad 0$ $\qquad \frac{15}{6}\times\frac{3}{7}$ $\quad \frac{45}{35}$ ^{9} $_{7}$

$(\times 3/7)$ $\quad 15/14 \quad 3/7 \quad 0 \quad 0 \quad \frac{\cancel{3}}{\cancel{2}}$ $\qquad \frac{35}{7}-\frac{9}{7}$

$(-) \quad (-) \quad (-) \quad (-) \quad (-)$

$5/2 \quad 0 \quad 0 \quad 1 \quad -\frac{\cancel{5}^{3}}{\cancel{2}} \rightarrow (2)$ $\qquad 7$

$\frac{15}{6}\times\frac{1}{7}$

$\frac{15}{\ } = 15/42$

	c_j		1	2	3	-1
C_B	y_B	x_B	x_1	x_2	x_3	x_4
2	x_2	$5/2$	0	1	0	$1/6$
3	x_3	$5/2$	0	0	1	$-3/6$
1	x_1	$5/2$	1	0	0	$7/6$

$z_j = C_B x_i$ = 15 · 1 2 3 0

$z_j - c_j$ 0 0 0 1

optium $z^* = 15$

subject to the constraints

$$z^* = x_1 + 2x_2 + 3x_3$$

$$x_1 = 5/2, \; x_2 = 5/2, \; x_3 = 5/2$$

$$z^* = 5/2 + 2(5/2) + 3(5/2)$$

$$= 5/2 + 10/2 + 15/2$$

$$z^* = \frac{30}{2}$$

$$\boxed{z^* = 15}$$

2. min z = $4x_1 + 6x_2 + 18x_3$

subject.

$x_1 + 3x_2 \geq 3$

$x_2 + 2x_3 \geq 5$ & $x_1, x_2\ x_3 \geq 0$

Solu:-

Standard form of the L.P.P

$\min z = 4x_1 + 6x_2 + 18x_3 + 0S_1 + 0S_2$

$x_1 + 3x_2 - S_1 = 3$

$x_2 + 2x_3 - S_2 = 5$

Dual of L.P.P

$\max z = 3w_1 + 5w_2$

$w_1 + \overset{0w_2}{\cancel{6w_2}} \leq 4$

$3w_1 + w_2 \leq 6$

$0w_1 + 2w_2 \leq 18$

---

3 min z $= 3x_1 - 2x_2 + 4x_3$

subject to construct.

$3x_1 + 5x_2 + 4x_3 \geq 7$

$6x_1 + x_2 + 3x_3 \geq 4$

$7x_1 - 2x_2 - x_3 \leq 10$

$x_1 - 2x_2 + 5x_3 \geq 3$

$4x_1 + 7x_2 - 2x_3 \geq 2$.

Solu:. Standard form of the L.P.P

$\max z = 3x_1 - 2x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3 + 0S_4$

$+ 0S_5$

$$3x_1 + 5x_2 + 4x_3 - s_1 = 7$$

$$6x_1 + x_2 + 3x_3 - s_2 = 4$$

$$7x_1 + 2x_2 - x_3 + s_3 = 10$$

$$x_1 - 2x_2 + 5x_3 - s_4 = 3$$

$$4x_1 + 7x_2 - 2x_3 - s_5 = 2$$

Dual of L.P.P.

$$\max z = 7w_1 + 4w_2 + 10w_3 + 3w_4 + 2w_5$$

$$3w_1 + 6w_2 + 7w_3 + w_4 + 4w_5 \leq 3$$

$$5w_1 + w_2 - 2w_3 + 2w_4 + 7w_5 \leq -2$$

$$4w_1 + 3w_2 - w_3 + 5w_4 - 2w_5 \leq 4$$

$$\left.\begin{array}{l} -w_1 + 0w_2 + 0w_3 + 0w_4 + 0w_5 \\ 0w_1 - w_2 + 0w_3 + 0w_4 + 0w_5 \\ 0w_1 + 0w_2 + w_3 + 0w_4 + 0w_5 \\ 0w_1 + 0w_2 + 0w_3 - w_4 + 0w_5 \\ 0w_1 + 0w_2 + 0w_3 + 0w_4 - w_5 \end{array}\right\} \leq 0$$

---

1. we duality to solve the L.P.P

$$\max z = 2x_1 + x_2$$

subject to construct

$$x_1 + 2x_2 \leq 10$$

$$x_1 + x_2 \leq 6$$

$$x_1 - x_2 \leq 2$$

$$x_1 - 2x_2 \leq 1$$

$$x_1, x_2 \geq 0$$

Soln:-

$\max z = 2x_1 + x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4$

$x_1 + 2x_2 + S_1 = 10 , \quad x_1 + x_2 + S_2 = 6$

$x_1 + x_2 + S_3 = 2 , \quad x_1 - 2x_2 \overset{+S_4}{=} 1 ,$

Dual OF L.P.P

$\min z = 10w_1 + 6w_2 + 2w_3 + w_4$

Sub to constraints

$w_1 + w_2 + w_3 + w_4 \overset{S_1 + A_1}{\geq} 2$

$2w_1 + 2w_2 + w_3 - 2w_4 \overset{S_2 + A_2}{\geq} 1$

	c_j		-10	-6	-2	-1	0	0	$-M$	$-M$	x_B/x_1
C_B	Y_B	x_B	x_1	x_2	x_3	x_4	S_1	S_2	A_1	A_2	
$-M$	A_1	2	1	1	1	1	-1	0	1	0	$2/1 = 2$
$-M$	A_2	1	$\textcircled{2}$	1	-1	-2	0	-1	0	1	$1/2 = 0.5$

$z_j = C_B x_i = (-3M) \; -3M \; -2M \; 0 \; M \; M \; M \; -M \; -M$ $\boxed{\min = 0.}$

$z_j - c_j \quad -3M+10 \; -2M+6 \; 2 \; M+1 \; M \; M \; 0 \; 0$

$\boxed{\text{most negative} -3M}$

$-1 \quad 2 \quad -1 \quad -1 \quad -2 \quad 0 \quad -1 \quad 0 \quad 1$

$(\div 2) \quad \frac{1}{2} \quad 1 \quad \frac{1}{2} \quad -\frac{1}{2} \quad -1 \quad 0 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2} \quad \rightarrow (2)$

$2 \quad 1 \quad 1 \quad +1 \quad + \quad -0 \quad -0 \quad 0 \quad 0$

$\frac{1}{2} \quad 1 \quad \frac{1}{2} \quad -\frac{1}{2} \quad -1 \quad 0 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2}$

$(-) \quad (-) \quad (-) \quad (+) \quad (+) \quad (-) \quad (+) \quad (-) \quad (-)$

$\frac{3}{2} \quad 0 \quad \frac{1}{2} \quad \frac{3}{2} \quad 2 \quad 0 \quad +\frac{1}{2} \quad 0 \quad -\frac{1}{2} \quad \rightarrow (1)$

$\frac{1}{2}$

$\frac{2-1}{2} \; \frac{1}{2}$

$\frac{3}{2}$

	c_j		-10	-6	-2	-1	0	0	$-M$	$-M$
C_B	Y_B	x_B	x_1	x_2	x_3	x_4	S_1	S_2	A_1	A_2
$-M$	A_1	$\frac{1}{2}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	-1	0	$-\frac{1}{2}$	0	$\frac{1}{2}$
-10	x_1	$\frac{3}{2}$	0	$\frac{1}{2}$	$\frac{3}{2}$	2	1	$\frac{1}{2}$	1	0

$\frac{1}{2} - \frac{30}{2}$

-30

$z_j = C_B x_i \quad -M$

	cj		-10	-6	-2	-1	0	0	-M	-M
CB	yB	wB	x_1	x_2	x_3	x_4	s_1	s_2	A_1	A_2
-M	A_1	5/2	0	1/2	$\boxed{5/2}$	2	-1	1/2	1	-1/2
-10	x_1	1/2	1	1/2	-1/2	-1	0	-1/2	0	1/2

Zj = CB xi $\frac{-5M-5}{2}$ -10 $\frac{-M}{2}-5$ $\frac{-5M+5}{2}$ $-2M+10$ M $-\frac{M}{2}+5$ $-M$ $\frac{-M}{2}$

$-2M+11$ M $-\frac{M}{2}+5$ 0 $\frac{M}{2}-5$

0 $\frac{-M+2}{2}$ $\frac{-5M+4}{2}$

5/2 0 1/2 5/2 2 -1 1/2 1 -1/2

(÷5/2) 1 0 1/5 1 4/5 -2/5 1/5 2/5 -1/5 → (1)

1/2 1 1/2 -1/2 -1 0 -1/2 0 1/2

(×1/2) 1/2 0 1/6 1/2 2/5 -1/5 1/6 1/5 -1/6

1 1 2/5 0 -1/5 -1/5 -1/5 1/5 1/5 → (2)

	cj		-10	-6	-2	-1	0	0	-M	-M
CB	yB	xB	x_1	x_2	x_3	x_4	s_1	s_2	A_1	A_2
-2	x_3		0	1/5	1	4/5	-2/5	1/5	2/5	-1/5
-10	x_1		1	$\boxed{2/5}$	0	-1/5	-1/5	-1/5	1/5	1/5

Zj = CB xi -12 -10 -22/5 -2 2/5 14/5 8/5 -14/5 -8/5

Zj - Cj $\cancel{0}$ 0 -4/5 0 5/5 14/5 8/5 $\frac{-14+M}{5}$ $\frac{-8+M}{5}$

1 1 2/5 0 -1/5 -1/5 -1/5 1/5 1/5

(×5/2) $\frac{5}{\cancel{2}}$ 5/2 1 0 -1/2 -1/2 -1/2 1/2 1/2 → (2)

$1 \quad 0 \quad 1/3 \quad 1 \quad 4/3 \quad -2/3 \quad 1/3 \quad 2/3 \quad -1/3$

$\times(-1/3) \quad -1/2 \quad 1/2 \quad -1/3 \quad 0 \quad 1/6 \quad 1/6 \quad 1/6 \quad -1/6 \quad -1/6$

$1/2 \quad -1/2 \quad 0 \quad 1 \quad 5/2 \quad -1/2 \quad 1/2 \quad 1/2 \quad -1/2 \rightarrow (1)$

	Cj →		-10	-6	-2	-1	0	0	-M	-M
MCB	y_B	x_B	x_1	x_2	x_3	x_4	S_1	S_2	A_1	A_2
-2	x_3	1/2	-1/2	0	1	5/2	-1/2	1/2	1/2	-1/2
-6	x_2	3/2	5/2	1	0	-1/2	-1/2	-1/2	1/2	1/2

$1 - 18/2$

$-2 \; 18/2$

$1/4$

$20/4 \; (1) \leftarrow$

$z_j = C_B x_i \quad -10 \quad -8 \quad -6 \quad -2 \quad 0 \quad 4 \quad 2 \quad -4 \quad -2$

$z_j - c_j \quad 2 \quad 0 \quad 0 \quad 1 \quad 4 \quad 2 \quad -4+M \quad -2+M$

optium $z^* = -10$

minimum $= -(-10)$

$= 10$

[illegible] $\quad -(4+M) \quad (-2+M)$

$(3/2) \quad x_1 - M \; , \; x_2 - M$

$x_1 - M = -M + 4$

$x_2 - M = -M + 2$

$x_1 = 4 \; , \; x_2 = 2$

$Z = 2x_1 + x_2$

$= 2(4) + 2$

$= 8 + 2$

$= 10$

Two-phase simplex method.

Use two-phase simplex method to maximize $z = 5x_1 - x_2$ subject to the constraints.

$2x_1 + x_2 \geq 2$, $x_1 + 3x_2 \leq 12$, $x_2 \leq 4$, $x_1, x_2 \geq 0$.

Solu :-

$\max z = 5x_1 - x_2 + 0s_1 + 0s_2 + 0s_3 - A_1$ ~~$+A_2$~~

$2x + x_2 - s_1 + A_1 = 2$

$x_1 + 3x_2 + s_2$ ~~$+A_2$~~ $= 12$, $x_2 + s_3 = 4$

phase-1 :-

	c_j		0	0	0	0	0	-1	~~0~~
C_B	y_B	x_B	x_1	x_2	s_1	s_2	s_3	A_1	~~A_2~~
-1	A_1	2	$\boxed{2}$	1	-1	0	0	1	~~0~~
0	s_2	12	1	3	0	+1	0	0	~~1~~
0	s_3	4	0	1	0	0	1	0	~~0~~

$Z_j = C_B x_i$ -2   -2   -1   +1   0   0   -1   ~~0~~

$z_j - c_j$ -2   -1   +1   0   0   0   ~~0~~

most negative -2, min=1

2 2 1 -1 0 0 1

(÷2) 1 1 1/2 -1/2 0 0 1/2 → (1)

2 1 3 0 1 0 0 $3-\frac{1}{2}$, $\frac{6-1}{2}$, = 5/2

1 1 1/2 -1/2 0 0 1/2

(-) (-) (-) (+) (-) (-) (-)

1 0 5/2 1/2 1 0 -1/2 → (2)

4 0 1 0 0 0 1 0 → (3)

	c_j		0	0	0	0	0	-1
C_B	y_B	x_B	x_1	x_2	s_1	s_2	s_3	A_1
0	x_1	1	1	1/2	-1/2	0	0	1/2
0	s_2	1	0	5/2	1/2	1	0	-1/2
0	s_3	4	0	1	0	0	1	0

x_B/x_1

2/2 = 1

2/1 = 2

4/0 = ∞

$z_j = C_B x_i$ 0 0 0 0 0 0 0

$z_j - c_j$ 0 0 0 0 0 1

<u>phase - 2</u> $z = 3x_1 - x_2 + s_1 + s_2 + s_3$.

	c_j		3	-1	0	0	0	
C_B	y_B	x_B	x_1	x_2	s_1	s_2	s_3	x_B/s_1
3	x_1	1	1	1/2	-1/2	0	0	$\frac{1}{1/2} = 1\times\frac{-2}{1} = \frac{-2}{1} = -2$
0	s_2	1	0	5/2	$\boxed{1/2}$	1	0	$\frac{1}{1/2} = 1\times\frac{2}{1} = \frac{2}{1} = 2$
0	s_3	4	0	1	0	0	1	4/0 = ∞

$z_j = C_B x_i$ 3 3 3/2 -3/2 0 0 min = 2

$z_j - c_j$ 0 5/2 -3/2 0 0

most negative $-\frac{5}{2}$

$1 \quad 0 \quad \frac{5}{2} \quad \frac{1}{2} \quad 1 \quad 0$

$(\div \frac{1}{2})$ $\overline{2 \quad 0 \quad 5 \quad 1 \quad 2 \quad 0} \rightarrow (2)$

$1 \quad 1 \quad \frac{1}{2} \quad -\frac{1}{2} \quad 0 \quad 0$

$1 \quad 0 \quad \frac{5}{2} \quad \frac{1}{2} \quad 1 \quad 0$

$2 \quad 1 \quad \cancel{\frac{5}{2}}3 \quad 0 \quad 1 \quad 0 \rightarrow (1)$

$4 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \rightarrow (3)$

	c_j		3	-1	0	0	0
C_B	y_B	x_B	x_1	x_2	s_1	s_2	s_3
3	x_1	2	1	3	0	1	0
0	s_1	2	0	5	1	2	0
0	s_3	4	0	1	0	0	1

$z_j = C_B x_i$ 6 3 9 0 3 0

$z_j - c_j$ 0 10 0 3 0

optium $Z^* = 6$

$x_1 = 2, x_2 = 0$

subject to constraint :-

$\max Z = 3x_1 - x_2$

$= 3(2) - 0$

$Z^* = 6$

$\frac{5}{2}$

$\times \frac{1}{2}$

$\frac{2 \times \frac{1}{2}}{1}$

$= -2$

$\frac{1}{2}$

$\frac{5}{2} \frac{1-5}{2}$

$+\frac{1}{2}$

$= \frac{0}{2}$

$\frac{1}{\frac{1}{2}} \times \frac{2}{1} =$

$\frac{5}{2} \times \frac{2}{1} =$

$\frac{1}{2} \times \frac{2}{1} = 1$

$1 \times \frac{2}{1} = \frac{2}{1}$

$\frac{\frac{1}{2}}{\frac{1}{2}}$

$-\frac{1}{2} + 1$

$\frac{-1+2}{2}$

$-\frac{1}{2} \times \frac{2}{2} \quad -1-2$

$\frac{-1+1}{2}$

2. Consider the L.P.P

$$\max z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

subject to $x_1 + x_2 + x_3 = 4$

$x_1 + 4x_2 + x_4 = 8$

$x_1, x_2, x_3, x_4 \geq 0$

By using x_3 and x_4 as the starting variable The optium table is given by

Basis	solution	x_1	x_2	x_3	x_4
x_3	2	3/4	0	1	-1/4
x_2	2	1/4	1	0	1/4
z	16	2	0	0	3

write the dual problem and Find its solution From the optium primal table :

<u>solu</u> :-

The dual problem is

$$\min z = 4w_1 + 8w_2$$

subject to

$w_1 + w_2 \geq 2$

$w_1 + 4w_2 \geq 4$

$w_1 \geq 4$

$w_2 \geq -3$

starting primal variable	x_3	x_4
corresponding net evalution	0	3
dual constraint associated with starting primal variable	$w_1 - 4$	$w_2 - (-3)$

$w_1 - 4 = 0$

$w_2 + 3 = 3$

$\therefore w_1 = 4$

$\therefore w_2 = 0$

$\min z = 4(4) + 8(0)$

$\min z = 16 = \text{Max } z$

---

3. Max $z = 5x_1 + 2x_2 + 5x_3$.

subject to $x_1 + 2x_2 + x_3 \leq a_1$, $3x_1 + 2x_3 \leq a_2$,

$x_1 + 4x_2 \leq a_3$

where $a_1\ a_2\ a_3$ are constants for specific value of $a_1\ a_2\ a_3$ the optium solution is

Basis	Solution	x_1	x_2	x_3	x_4	x_5	x_6
x_2	100	b_1	1	0	$1/2$	$-1/4$	0
x_3	c_3	b_2	0	1	6	$1/2$	0
x_6	20	b_3	0	0	-2	1	1
z	1550 $C_B x_B$	4	0	0	c_1	c_2	6

Determine

i) The value of $a_1\ a_2\ a_3$ that yield the given optium solution.

ii) The values of $b_1\ b_2\ b_3$ and $c_1\ c_2\ c_3$ in the optium.

iii) The optimal dual solution.

Solu:-

i) $b^{-1}b = x_B$

$$\Rightarrow \begin{bmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 100 \\ c_3 \\ 20 \end{bmatrix}$$

$$1/2\, a_1 - 1/4\, a_2 = 100$$

$$1/2\, a_2 = c_3$$

$$-2a_1 + a_2 + a_3 = 20$$

Also

$$Z = C_B x_B = 1350$$

$$1350 = 5c_3 + 200$$

$$5c_3 + 200 = 1350$$

$$5c_3 = 1350 - 200$$

$$\boxed{c_3 = 230}$$

$$1/2\, a_2 = 230$$

$$a_2 = 230 \times 2$$

$$\boxed{a_2 = 460}$$

$$1/2\, a_1 - \frac{1}{\cancel{4}_{2}} \times \overset{115}{\cancel{460}^{230}} = 100$$

$$1/2\, a_1 = 100 + 115$$

$$a_1 = 215 \times 2$$

$$\boxed{a_1 = 430}$$

$$-2(430) + 460 + a_3 = 20$$

$$-860 + 460 + a_3 = 20$$

$$-400 + a_3 = 20$$

$$a_3 = 20 + 400$$

$$\boxed{a_3 = 420}$$

ii) Z-row

$4 = CBY_1 - C_1$

$4 = 2b_1 + 5b_3 = 3$

$2b_1 + 5b_3 = 7$

$c_1 = CBY_4 - C_4$

$= 1 - 0 = 1$ $\boxed{c_1 = 1}$

$c_2' = CBY_5 - C_5$

$= -1/2 + 5/2 - 0 = 2$

$\boxed{c_2 = 2}$

$Z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

$x_1 + 2x_2 + x_3 + s_1 = 430$

$3x_1 + 0x_2 + 2x_3 + s_2 = 460$

$x_1 + 4x_2 + 0x_3 + s_3 = 420$

	c_j		3	2	5	0	0	0	
C_B	y_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3	x_B/x_3
0	s_1	430	1	2	1	1	0	0	$430/1 = 43$
0	s_2	460	3	0	$\textcircled{2}$	0	1	0	$460/2 = 2$
0	s_3	420	1	4	0	0	0	1	$420/0 =$
$z_j = CBx_i$			0	0	0	0	0	0	min
$z_j - c_j$			-3	-2	-5	0	0	0	

most negative = -5

(2) equ ÷ 2

$250 \quad 5/2 \quad 0 \quad 1 \quad 0 \quad 1/2 \quad 0 \rightarrow (2)$

$(3) \Rightarrow 420 \quad 1 \quad 4 \quad 0 \quad 0 \quad 0 \quad 1 \rightarrow (3)$

$(1) - (2)$

$450 \quad 1 \quad 2 \quad 1 \quad 1 \quad 0 \quad 0$

$250 \quad 5/2 \quad 0 \quad 1 \quad 0 \quad 1/2 \quad 0$

$(-) \quad (-) \quad (-) \quad (-) \quad (+) \quad (-) \quad (+)$

$200 \quad -1/2 \quad 2 \quad 0 \quad 1 \quad -1/2 \quad 0 \rightarrow (1)$

	C_j		3	2	5	0	0	0	
C_B	y_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3	x_B/x_2
0	S_1	200	$-1/2$	$\boxed{2}$	0	1	$-1/2$	0	$200/2=1$
5	x_3	250	$5/2$	0	1	0	$1/2$	0	$250/0=\infty$
0	S_3	420	1	4	0	0	0	1	$420/4=1$

$Z_j = C_B x_i \quad 1150 \quad 15/2 \quad 0 \quad 5 \quad 0 \quad 5/2 \quad 0$

$Z_j - C_j \quad 9/2 \quad -2 \quad 0 \quad 0 \quad 5/2 \quad 0$ most negative -2

$(1) \quad 200 \quad -1/2 \quad 2 \quad 0 \quad 1 \quad -1/2 \quad 0$

$\div 2) \quad 100 \quad -1/4 \quad 1 \quad 0 \quad 1/2 \quad -1/4 \quad 0 \rightarrow (1)$

$(3) \Rightarrow 420 \quad 1 \quad 4 \quad 0 \quad 0 \quad 0 \quad 1$

$400 \quad -1 \quad 4 \quad 0 \quad 2 \quad -1 \quad 0$

$(-) \quad (+) \quad (-) \quad (-) \quad (-) \quad (+) \quad (-)$

$20 \quad 2 \quad 0 \quad 0 \quad -2 \quad 1 \rightarrow (3)$

$(2) \rightarrow 250 \quad 5/2 \quad 0 \quad 1 \quad 0 \quad 1/2 \quad 0 \rightarrow (2)$

	cj		3	2	5	0	0	0
CB	yB	xB	x_1	x_2	x_3	s_1	s_2	s_3
2	x_2	100	$-1/4$	1	0	$1/2$	$-1/4$	0
5	x_3	230	$3/2$	0	1	0	$1/2$	0
0	s_3	20	2	0	0	-2	1	1
$z_j = C_B x_j$		1350	7	2	5	1	2	0
$z_j - c_j$			4	0	0	1	2	0

(ii) $b_1 = -1/4$, $b_2 = 3/2$, $b_3 = 2$

$c_1 = 1$, $c_2 = 2$, $c_3 = 0$

The dual problem is

min $z = w_1 + w_2 + w_3$

subject to con

$w_1 + 3w_2 + w_3 \geq 3$

$2w_2 + 0w_2 + 4w_3 \geq 2$

$w_1 + 2w_2 + 0w_3 \geq 5$

Starting primal variable	x_4	x_5	s_3
dual constraints	$w_1 - 0$	$w_2 - 0$	$w_3 - 0$
Net evaluation	c_1	c_2	0

$c_1 = w_1 - 0 \Rightarrow w_1 = c_1 = 1$

$c_2 = w_2 - 0 \Rightarrow w_2 = c_2 = 2$

$0 - w_3 = 0 \Rightarrow w_3 = 0$

ii) optium dual solution is

min $z = a_1 w_1 + a_2 w_2$

$= 430 \times 1 + 460 \times 2$

$= 430 + 920 = 1350$

III - unit - complete

(5 mark) **Theorem:-** Dual of the Dual is the primal.

Proof:-

Let the primal LPP be

$\max f(x) = z = cx$

sub to $Ax \leq b$

$x \geq 0$

By difn its dual is given by

$\min f(w) = w = b^T w$

sub to

$A^T w \geq c^T$

$w \geq 0$

The canonical form of the dual is given by

$\max z = f(w) = -b^T w$

sub to

$-A^T w \leq -c^T$

$w \geq 0$

Hence the dual of the dual is given by

$\min h(y) = (-c^T)^T y$

sub to $(-A^T)^T y \geq (-b^T)^T$

$y \geq 0$

i) $\max h(y) = -(-cy)$

sub to $-Ay \geq -b$

$y \geq 0$

ie) Max $h(y) = cy$

sub to $Ay \leq b$

$y \geq 0$

This L.PP that is the dual of the dual problem is same as the primal problem

Hence the result.

Unit-III

Solution of a Transportation Problem

2.8.1

1039. National oil company (NOC) has three refineries and four depots Transporatation cost per ton capacities and requirements are given below.

	D1	D2	D3	D4	ai capacity (tons)
R1	5	7	13	10	700
R2	8	6	14	13	400
R3	12	10	9	11	800
Requirement bj	200	600	700	400	

Determine optimum allocation of output.

Solution :-

North-west Corner Method :-

	D_1	D_2	D_3	D_4	
R_1	200 5	500 7	15	10	700 - 200 = 500
R_2	8	100 6	300 14	13	400-100=300
R_3	12	10	400 9	400 11	800 - 400 = 400
	200	600 600-500 =100	700 -300 =400	400	1900

$\sum_i a_i = \sum_j b_j = 1900$ there exists a feasible solution to the transporatation problem

first allocation is made in the cell (1,1)

$$x_{11} = \min(700, 200) = 200$$

second allocation is made in the cell (1,2)

$$x_{12} = \min(700-200, 600) = 500$$

Third allocation is made in the cell (2,2)

$$x_{22} = \min(600-500, 400) = 100$$

Fourth allocation is made in the cell (2,3)

$$x_{23} = \overset{\min}{}(400-100, 700) = 300$$

Fifth allocation is made in the cell (3,3)

$$x_{33} = \min(700-300, 800) = 400$$

Sixth allocation is made in the cell (3,4)

$$x_{34} = \min(800-400, 400) = 400$$

$\min z = 5 \times 200 + 7 \times 500 + 6 \times 100 + 14 \times 300 + 9 \times 400 + 11 \times 400$

$= 1000 + 3500 + 600 + 4200 + 3600 + 4400$

$\min z = 17,300$

Least. cost - Method

	D_1	D_2	D_3	D_4	
R_1	200 5	200 7	15	300 10	700 - 200 = 500 - 200 = 300
R_2	8	400 6	14	15	400
R_3	12	10	700 9	100 11	800 - 700 = 100
	200	600 -200	700	400 =100	

$\sum_i a_i = \sum_j b_j = 1900$

The first allocation is made in the cell (1,1)

$x_{11} = \min(700, 200) = 200$

$(1,2) \Rightarrow x_{12} = \min(600-400, 500) = 200$

$(1,3) \Rightarrow x_{13} = (500, -200, 400) = 300$

$(2,2) = x_{22} = (400, 600) = 400$

$(3,3) = x_{33} = (700, 800) = 700$

$(3,4) = x_{34} = (800-700, 400-300) = 100$

#17 $\min z = 5 \times 200 + 7 \times 200 + 10 \times 300 + 6 \times 400 + 9 \times 700 + 11 \times 100$

$= 1000 + 1400 + 3000 + 2400 + 6300 + 1100$

$\min z = 15,200$

100) obtain an initial basic feasible solution to the Following transporatation problem using the north-west corner rule:

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

Solu :-

	D	E	F	G	ai
A	200 | 11	50 | 13	17	14	250-200 = 50
B	16	175 | 18	125 | 14	10	300-175 = 125
C	21	24	150 | 13	10 | 250	400-150 = 250
bj	200	225 -50 =175	275 -125 =150	250	

$\sum_{i} ai = \sum_{j} bj = 950$

The First allocation is made in the cell (1,1)

$x_{11} = \min(250, 200) = 200$

The second allocation is made in the cell (1,2)

$x_{12} = \min(250-200, 225) = 50$

The Third allocation is made in the cell (2,2)

$x_{22} = \min(300, 225-50) = 175$

The Forurth allocation is made in the cell (2,3)

$x_{23} = \min(300-175, 275) = 125$

The Fifth allocation is made in the cell (3,3)

$x_{33} = \min(400, 275-125) = 150$

The sixth allocation is made in the cell (3,4)

$x_{34} = \min(400-150, 250) = 250$

$\min z = 11 \times 200 + 13 \times 50 + 18 \times 175 + 14 \times 125 + 13 \times 150 + 10 \times 250$

$= 2200 + 650 + 3150 + 1750 + 1950 + 2500$

$= 12200$

Least Cost - Method

	D	E	F	G	
A	200/ 11	50/ 15	17	14	250 - 200 = 50
B	16	50/ 18	14	250/ 10	500 - 250 = 50
C	21	125/ 24	275/ 13	10	400 - 275 = 125
	200	225 -50 =175 -125=50	275 -50 =225	250	

	D	E	F	G
A	200/ 11	50/ 15	17	14
B	16	50/ 18	14	250/ 10
C	21	125/ 24	275/ 13	10

$\Sigma a_i = \Sigma b_i = 950$

The First allocation is made in the cell (1,1)

$$x_{11} = \min(250, 200) = 200$$

The second allocation is made in the cell (1,2)

$$x_{12} = \min(250-200, 225) = 50$$

The Third allocation is made in the cell (2,2)

$$x_{22} = \min(500-250, 225) = 50$$

The Forth allocation is made in the cell (2,

$$x_{24} = \min(250, 300) = 250$$

The Fifth allocation is made in the cell (3,2)

$$x_{32} = (400-275) 175) = 125$$

The sixth allocation is made in the cell (3,3

$$x_{33} = (400, 275) = 275$$

$$\min z = 11 \times 200 + 13 \times 50 + 18 \times 50 + 10 \times 250 + 24 \times 125 + 13 \times 275$$

$$= 2200 + 650 + 900 + 2500 + 3000 + 3575$$

$$= 12,825$$

1005. determine an Initial basic Feasible soln to the Following transportation

	D_1	D_2	D_3	D_4	Availability
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
Demand	16	18	31	25	

Solution :-

North-west Corner Method.

	D_1	D_2	D_3	D_4	
O_1	5 $\boxed{16}$	3 $\boxed{3}$	6	2	19-16 = 3
O_2	4	7 $\boxed{15}$	9 $\boxed{22}$	1	37-15 = 22
O_3	3	4	7 $\boxed{9}$	5 $\boxed{25}$	34-9 = 25
	16	18-3 15	31-22 =9	25	

$$\sum_i a_i = \sum_j b_j = 90$$

The First allocation is made in the cell (1,1)

$$x_{11} = \min(19, 16) = 16$$

The second allocation is made in the cell (1,2)

$$x_{12} = \min(19-16, 18) = 3$$

The third allocation is made in the cell (2, 2)

$$x_{22} = \min(18-3, 37) = 15$$

The foruth allocation is made in the cell (2, 3)

$$x_{23} = \min(37-15, 31) = 22$$

The fifth allocation is made in the cell (3, 3)

$$x_{33} = \min(31-22, 34) = 9$$

The sixth allocation is made in the cell (3, 4)

$$x_{34} = \min(34-9, 25) = 25$$

min $z = 16\times5 + 3\times3 + 15\times7 + 9\times22 + 9\times7 + 5\times25$

$= 80 + 9 + 105 + 198 + 63 + 125$

$= 580.$

Least - cost method

	D_1	D_2	D_3	D_4	
O_1	5	$\boxed{18}$ 3	$\boxed{1}$ 6	2	19-18 = 1
O_2	4	7	$\boxed{12}$ 9	$\boxed{25}$ 1	37 - 25 = 12
O_3	$\boxed{16}$ 3	4	$\boxed{18}$ 7	5	34-16 = 18
	16	18	31-1 = 30-18 = 12	25	

$\Sigma a_i = \Sigma b_j = 90$

	D1	D2	D3	D4	
O1	5	3 $\lfloor 18$	6 $\lfloor 1$	2	1
O2	4	7	9 $\lfloor 12$	1 $\lfloor 25$	12
O3	$\lfloor 16$ 3	4	$\lfloor 18$ 7	5	18
	16	18	12	25	

$$\min z = 3\times18 + 1\times6 + 9\times12 + 1\times25 + 16\times3 + 7\times18$$

$$= 54 + 6 + 108 + 25 + 48 + 126$$

$$= 367$$

2. determine an initial basic feasible solu to the following transportation

Destination

Source	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Requirement	60	40	30	110	240

Solu:-

North-west corner Method

	D_1	D_2	D_3	D_4	
1	$\boxed{60}$ 20	$\boxed{40}$ 22	$\boxed{20}$ 17	4	120-60=60-40=20
2	24	37	$\boxed{10}$ 9	$\boxed{60}$ 7	70-10=60
3	32	37	20	$\boxed{50}$ 15	50
	60	40	30 -20 =10	110-60 =50	

$$\sum_i a_i = \sum_j b_j = 240$$

The First allocation is made in the cell (1,1)

$$x_{11} = \min(120, 60) = 60$$

The second allocation is made in the cell (1,2)

$$x_{12} = \min(120-60, 40) = 40$$

The Third allocation is made in the cell (1,3)

$$x_{13} = \min(60-40, 30) = 20$$

The Fourth allocation is made in the cell (2,3)

$$x_{23} = \min(30-20, 70) = 10$$

The Fifth allocation is made in the cell (2,4)

$$x_{24} = \min(70-10, 110) = 60$$

The sixth allocation is made in the cell (3,4)

$$x_{34} = \min(110-60, 50) = 50$$

$$\min z = 20\times60 + 22\times40 + 17\times20 + 9\times10 + 7\times60 + 15\times50$$

$$= 1200 + 880 + 340 + 90 + 420 + 750$$

$$\min z = 3680$$

Least Coost - Method

	D1	D2	D3	D4	
1	$\boxed{110}$ 20	22	17	$\boxed{110}$ 4	120-110=10
2	$\boxed{40}$ 24	37	$\boxed{30}$ 9	7	70-30=40
3	$\boxed{10}$ 32	$\boxed{40}$ 37	20	15	50-10=40
	60-10 =50-40 =10	40	30	110 =10	

$$\Sigma a_i = \Sigma b_j = 240$$

	D1	D2	D3	D4	
1	$\boxed{10}$ 20	22	17	$\boxed{110}$ 4	10
2	$\boxed{40}$ 24	37	9	$\boxed{30}$ 7	40
3	$\boxed{10}$ 32	$\boxed{40}$ 37	20	15	40
	10	40	30	110	

$$\min Z = 20\times10 + 4\times110 + 24\times40 + 9\times30 + 32\times10 + 37\times40.$$

$$= 200 + 440 + 960 + 270 + 320 + 1480$$

$$= 3670$$

---

3. determine an Initial basic feasible solu to the following transportation

Stores

ware house	I	II	III	IV	Availability
A	5	1	3	3	34
B	3	3	5	4	15
C	6	4	4	3	12
D	4	-1	4	2	19
Requirement	21	25	17	17	80

Solu:-

North West Corner - Method.

	I	II	III	IV	
A	5 $\boxed{21}$	1 $\boxed{13}$	3	3	34 - 21 = 13
B	3	3 $\boxed{12}$	5 $\boxed{3}$	4	15 - 12 = 3
C	6	4	4 $\boxed{12}$	3	12
D	4	-1	4 $\boxed{2}$	2 $\boxed{17}$	19 - 2 = 17
	21	25 -13 =12	17-3 14-12 =2	17.	

$\sum_i a_i = \sum_j b_j = 80$

The First allocation is made in the cell (1,1)

$x_{11} = \min(34, 21) = 21$

The second allocation is made in the cell (1,2)

$x_{12} = \min(34-21, 25) = 13$

The Third allocation is made in the cell (2,2)

$x_{22} = \min(25-13, 15) = 12$

The Fourth allocation is made in the cell (2,3)

$x_{23} = \min(15-12, 17) = 3$

The Fifth allocation is made in the cell (3,3)

$x_{33} = \min(12, 14) = 12$

The sixth allocation is made in the cell (4,3)

$x_{43} = \min(14-12, 19) = 2$

The seventh allocation is made in the cell (4,4)

$x_{44} = \min(19-2, 17) = 17$

$\min z = 5\times21 + 1\times15 + 12\times3 + 5\times3 + 12\times4 + 4\times2 + 17\times2$

$= 105 + 15 + 36 + 15 + 48 + 8 + 34$

$\min z = 259$

Least cost - Method.

	I	II	III	IV	
A	5	1 $\boxed{6}$	3 $\boxed{17}$	3 $\boxed{11}$	34 - 6 = 28 - 17 = 11
B	3 $\boxed{15}$	3	5	4	15
C	6 $\boxed{6}$	4	4	3 $\boxed{6}$	12 - 6 = 6
D	4	-1 $\boxed{19}$	4	2	19
	21-15 =6	25-19 = 6	17	17-11 =6	

	I	II	III	IV	
A	5	1 $\boxed{6}$	3 $\boxed{17}$	3 $\boxed{11}$	34
B	3 $\boxed{15}$	3	5	4	15
C	6 $\boxed{6}$	4	~~8~~ 4	3 $\boxed{6}$	12
D	~~2~~ 4	-1 $\boxed{19}$	4	[illegible] 2	19
	21	25	17	17	

$\Sigma a_i = \Sigma b_i = 80$

~~$\min z = 1\times25 + 3\times9 + 4\times8 + 6\times4 + 2\times17 + 4\times2 + 15\times3$~~

~~$= 25 + 27 + 32 + 24 + 34 + 8 + 45$~~

~~$\min z = 195$~~

$\min z = 1\times6 + 3\times17 + 3\times11 + 15\times3 + 6\times6 + 6\times3 + (-1)\times19 +$

$= 6 + 51 + 33 + 45 + 36 + 18 + (-19)$

$= 170$

---

ware house

4.

Factories	W_1	W_2	W_3	W_4	W_5	Availability
F_1	20	28	32	55	70	50
F_2	48	36	40	44	48	100
F_3	55	55	22	45	48	150
Requirment	100	70	50	40	40	300

Solu:-

North-west Method.

	W_1	W_2	W_3	W_4	W_5	
F_1	50 20	28	32	55	70	50
F_2	50 48	50 36	40	44	48	100-50=50
F_3	35	20 55	50 22	40 45	40 48	150-20=130 -50 =80-40 =40
	100 -50 =50	70-50 =20	50	40	40	

$\sum_i a_i = \sum_j b_j = 300$

The first allocation is made in the cell (1,1)

$x_{11} = \min(50, 100) = 50$

The second allocation is made in the cell (2,1)

$x_{21} = \min(100-50, 100) = 50$

The third allocation is made in the cell (2,2)

$x_{22} = \min(100-50, 70) = 50$

The fourth allocation is made in the cell (3,2)

$x_{32} = \min(70-50, 150) = 20$

The fifth allocation is made in the cell (3,3)

$x_{33} = \min(\overset{150-20,}{\cancel{180}}, 50) = 50$

The sixth allocation is made in the cell (3,4)

$x_{34} = \min(\overset{150-50}{80}, 40) = 40$

The seventh allocation is made in the cell (3,5)

$x_{35} = \min(80-40, 40) = 40$

$\min z = 50\times20 + 50\times48 + 50\times36 + 20\times55 + 50\times22 +$

$45\times40 + 40\times48$

$= 1000 + 2400 + 1800 + 1100 + 1100 + 1800 + 1920$

$= 11{,}120$

Least - Cost Method.

	W_1	W_2	W_3	W_4	W_5	
F_1	$\overset{\boxed{50}}{20}$	28	32	55	70	50
F_2	78	$\overset{\boxed{70}}{36}$	40	$\overset{\boxed{30}}{44}$	48	100-70 =30
F_3	$\overset{\boxed{50}}{35}$	55	$\overset{\boxed{50}}{22}$	$\overset{\boxed{10}}{45}$	$\overset{\boxed{40}}{48}$	150-50 =100-50 =50-10 =40
	100-50 =50	70	50	40 -30 =10	40	

$\Sigma a_i = \Sigma b_i = 300$

$\min z = 20\times50 + 36\times70 + 44\times30 + 35\times50 + 22\times50 + 45\times1$

$+ 48\times40$

$= 1000 + 2520 + 1320 + 1750 + 1100 + 450 + 19$

$= 10{,}060$

5.

	M_1	M_2	M_3	M_4	supply
F_1	4	6	8	13	1500
F_2	13			8	700
F_3	4	4	10	15	300
F_4	9	11	15	15	500
Requirement	250	1550	1050	200	

Soln:-

	M_1	M_2	M_3	M_4	M_5	
F_1	250 4	250 6	8	15	0	500 - 250 = 250
F_2	15	100 11	1.050 10	200 8	398.95 0	700 - 100 = 600 - 1.050 = 598.95 200 - 398.9
F_3	4	4	10	15	300 0	300
F_4	9	11	15	3	500 0	1500
	250	350 - 250 = 100	1.050	200	1198.95 398.95 = 800 - 300 = 500	

$\Sigma ai = \Sigma bj$

The first allocation is made in the cell (1,1)

$x_{11} = \min(500, 250) = 250$

$x_{12} = \min(500 - 250, 350) = 250$

$x_{22} = \min(350 - 250, 700) = 100$

$x_{23} = \min(700 - 100, 1.050) = 1.050$

$x_{24} = \min(600 - 1.050, 200) = 200$

$x_{25} = \min(598.95 - 200, 1198.95)$

$= 398.95$

$x_{35} = \min(1198.95 - 398.95, 300) = 300$

$x_{45} = \min(800 - 300, 500) = 500$

1. VOgal Method

	D	E	F	G.		
A	5	3	6	2	250	(1)
B	4	7	9	250 1	300-250 = 50	(3)
C	3	4	7	5	400	(1)
	200 (1)	225 (1)	275 (1)	250 (1)		

5	3	6	250	(2)
50 4	7	9	50	(3)
3	4	7	400	(1)
200-50 =150 (1)	225 (1)	275 (1)		

5	3	6	250	(2)
150 3	4	7	400-150 =250	(1)
150 (2)	225 (1)	275 (1)		

225 3	6	250-225 =25	(3)
4	7	250	(3)
225 (1)	275 (1)		

25 6	25
7	250
275-25 250	

250 7	250
250	

	w_1	w_2	w_3	w_4	w_5		
F_1	20	28	32	55	70	50	(8)
F_2	48	36	40	44	40 25	100-40 =60	(11)
F_3	35	55	22	45	48	150	(13)
	100	70	50	40	40		
	(15)	(8)	(10)	(1)	(23)		

50 20	28	32	55	50	(8)
48	36	40	44	60	(4)
35	55	22	45	150	(13)
100-50 =50	70	50	40		
(15)	(8)	(10)	(11)		

48	60 36	40	44	60	(4)
35	55	22	45	150	(13)
50	70-60 =10	50	40		
(13)	(19)	(18)	(1)		

35	55	50 22	45	150-50 =100
50	10	50	40	

50 35	55	45	100-50 =50 (10)
50	10	40	

55	40 45	50-40 =10 (10)
10	40	

10 55	10
10	

Min $Z = 40\times25 + 50\times25 + 60\times36 + 50\times22 + 50\times35 + 40\times45$
$+ 10\times55 = 9{,}360.$

3.

	I	II	III	IV		
A	5	1	3	3	34	(2)
B	3	3	5	4	15	(0)
C	6	4	4	3	12	(1)
D	4	$\boxed{19}$ -1	4	2	19	(3)
	21	25-19 =6	17	17		
	(1)	(2)	(1)	(1)		

5	1	3	3	34	(2)
$\boxed{15}$ 3	3	5	4	15	(0)
6	4	4	3	12	(1)
21-6 =6	6	17	17		
(2)	(2)	(1)	(0)		

5	$\boxed{6}$ 1	3	3	34-6 =28	(2)
6	4	4	3	12	(1)
6	6	17	17		
(1)	(3)	(1)	(0)		

$\boxed{6}$ 5	3	3	28 - 6 = 22	(0)
6	4	3	12	(1)
6	17	17		
(1)	(1)	(0)		

$\boxed{17}$ 3	3	22-17 = 5	(0)
4	3	12	(1)
17	17		
(1)	(0)		

$\boxed{5}$ 3	5
3	12
17-5 =12	
(0)	

3 $\boxed{12}$	12
12	

$$\min z = 19\times(-1) + 15\times3 + 6\times1 + 6\times5 + 17\times3 + 5\times3 + 12\times3$$

$$z = 164$$

TRANSPORATATION ALGORITHM (MODI Method)

1011 Given $x_{13}=50$ units, $x_{14}=20$ units $x_{21}=55$ unit $x_{31}=30$ units $x_{32}=35$ units and $x_{34}=25$ uni $x_{31}=30$ units if is an optimal solution to the transportatio problem.

$$\begin{array}{l|cccc|l} & & & & & \text{Available units} \\ & 6 & 1 & 9 & 3 & 70 \\ & 11 & 5 & 2 & 8 & 55 \\ & 10 & 12 & 4 & 7 & 90 \\ \hline \text{Requirement units} & 85 & 55 & 50 & 45 & \end{array}$$

If not modify is to obtain a better feasible solution

Solution:-

6	1	50 9	20 3	70	u_1
55 11	5	2	8	55	u_2
30 10	35 12	4	25 7	90	u_3
85	55	50	45		
v_1	v_2	v_3	v_4		

The equation $u_i+v_j=c_{ij}$ assume that $u_3=0$

$u_3+v_1=c_{31}$

$0+v_1=10$

$\boxed{v_1=10}$

$u_3+v_2=c_{32}$

$0+v_2=12$

$\boxed{v_2=12}$

$u_3+v_4=c_{34}$

$0+v_4=7$

$\boxed{v_4=7}$

step-3.

$u_1 + v_4 = c_{14}$

$u_1 + 7 = 3$

$u_1 = 3 - 7$

$\boxed{u_1 = -4}$

$u_2 + v_1 = c_{21}$

$u_2 + 10 = 11$

$u_2 = 11 - 10$

$\boxed{u_2 = 1}$

$u_1 + v_3 = c_{13}$

$-4 + v_3 = 9$

$v_3 = 9 + 4$

$\boxed{v_3 = 13}$

$\boxed{u_3 = 0}$

step-4

$z_{ij} - c_{ij} = u_i + v_j - c_{ij}$

$z_{11} - c_{11} = u_1 + v_1 - c_{11} = -4 + 10 - 6 = 0$

$z_{12} - c_{12} = u_1 + v_2 - c_{12} = -4 + 12 - 1 = -5 + 12 = 7$

$z_{22} - c_{22} = u_2 + v_2 - c_{22} = 1 + 12 - 5 = 13 - 5 = 8$

$z_{23} - c_{23} = u_2 + v_3 - c_{23} = 1 + 13 - 2 = 12$

$z_{24} - c_{24} = u_2 + v_4 - c_{24} = 1 + 7 - 8 = 0$

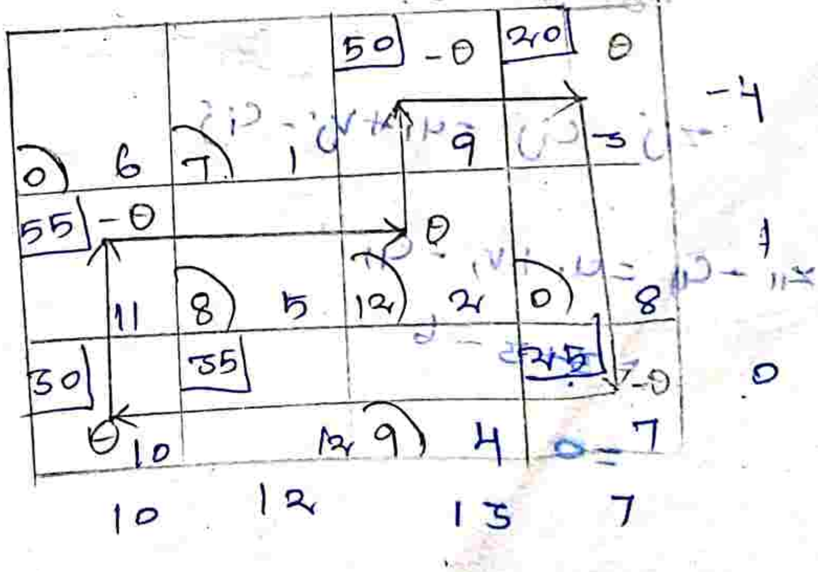
$z_{33} - c_{33} = u_3 + v_3 - c_{33} = 0 + 13 - 4 = 9$

step-5 :

Since all $z_{ij} - c_{ij} \geq 0$ solution is no

The current basic feasible

optium :



					u_i
	(0) 6	(7) 1	50 −θ 9	20 θ 3	−4
	55 −θ 11	(8) 5	θ (12) 2	(0) 8	1
	30 θ 10	35 12	(9) 4	25 −θ 7	0
v_j	10	12	13	7	

All non basic is not ≤ 0

3 is ≥ 0 see. we form Loop

$\theta = \min\{25, 55, 50\}$

$\theta = 25$

6	1	25 9	45 3	u_1
30 11	5	25 2	8	u_2
55 10	35 12	4	7	u_3
v_1	v_2	v_3	v_4	

$v_4 = 0$ $\qquad u_i + v_j = c_{ij}$

$u_1 + v_4 = c_{14}$

$u_1 + 0 = 3$

$\boxed{u_1 = 3}$

$u_2 + v_4 = c_{24}$

$u_2 + 0 = 8$

$\boxed{u_2 = 8}$

$u_3 + v_4 = c_{34}$

$u_3 + 0 = 7$

$\boxed{u_3 = 7}$

$v_1 + u_1 = c_{11} = v_1 + 3 = 6$, $v_1 = 6 - 3$, $\boxed{v_1 = 3}$

$v_2 + u_2 + c_{22} = v_2 + 8 = 5$, $v_2 = 5 - 8$, $\boxed{v_2 = -3}$

$v_3 + u_3 = c_{33} = v_3 + 7 = 4$, $v_3 = 4 - 7$, $\boxed{v_3 = -3}$

$z_{ij} - c_{ij} = u_i + v_j - c_{ij}$

$z_{11} - c_{11} = u_1 + v_1 - c_{11}$

$= 3 + 3 - 6$

$Z_{12} - C_{12} = u_1 + V_2 - C_{12} = 3 - 3 - 1 = -1$

$Z_{22} - C_{22} = u_2 + V_2 - C_{22}$

$= 8 - 3 - 5$

$= 0$

$Z_{24} - C_{24} = u_2 + V_4 - C_{24}$

$= 8 + 0 - 8$

$= 0$

$Z_{33} - C_{33} = u_3 + V_3 - C_{33}$

$= 7 - 3 - 4 = 0$

$Z_{34} - C_{34} = u_3 + V_4 - C_{34}$

$= 7 + 0 - 7$

$= 0$

(0) 6	(-1) 1	$\boxed{25}$ 9	$\boxed{45}$ 3	3
$\boxed{30}$ 11	(0) 5	$\boxed{25}$ 2	(0) 8	8
$\boxed{55}$ 10	$\boxed{35}$ 12	(0) 4	(0) 7	7
3	-3	-3	0	

The optium soln $x_{21} = 30$, $x_{23} = 25$,

$x_{13} = 25$, $x_{14} = 45$,

$x_{31} = 55$, $x_{32} = 35$.

$\min = 30(11) + 25(2) + 25(9) + 45(3) + 55(10) +$

$35(12)$

$= 1710$

1010. Find the starting in the following transporation problem by vogel's Approximation method.

Also obtain the optium solution

	D_1	D_2	D_3	D_4	supply
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
Demand.	3	3	2	2	

<u>solution</u> :-

$\boxed{3}$ 3	7	6	$\boxed{2}$ 4	5-3 =2	(1) (1) (1)	u_1
2	4	$\boxed{2}$ 3	$\boxed{\varepsilon}$ 2	2 (0)	— —	u_2
4	$\boxed{3}$ 3	8	$\boxed{\varepsilon}$ 5	3	(1) (1) —	u_3
3	3	2	2			
(1)	(1)	(3)	(2)			
(1)	(4)	—	(1)			
v_1	v_2	v_3	v_4			

$u_i = (i=1,2,3)$

$v_j \, (j=1,2,3,4)$

$u_i+v_j = c_{ij}$

$v_4 = 0$

Step-3

$u_i+v_j = c_{ij}$

$u_1+v_4 = c_{14}$

$u_1+0 = 4$

$\boxed{u_1 = 4}$

$u_2+v_4 = c_{24}$

$u_2+0 = 2$

$\boxed{u_2 = 2}$

$u_3+v_4 = c_{34}$

$u_3+0 = 5$

$\boxed{u_3 = 5}$

Given u_1 u_2 and u_3 valuese of v_1 v_2 . v_3 can be ~~made~~ calculated as shown below.

$u_1+v_1 = c_{11}$

$4+v_1 = 3$

$v_1 = 3-4$

$\boxed{v_1 = -1}$

$u_2+v_3 = c_{23}$

$2+v_3 = 3$

$v_3 = 3-2$

$\boxed{v_3 = 1}$

$u_3+v_2 = c_{32}$

$5+v_2 = 3$

$v_2 = 3-5$

$\boxed{v_2 = -2}$

Step-4

The net evalutions for each of the unoccupied calls are now determined.

$z_{ij} - c_{ij} = u_i+v_j - c_{ij}$

$z_{12} - c_{12} = u_1+v_2 - c_{12}$

$= 4-2-7$

$= 4-9$

$\boxed{z_{12}-c_{12} = -5}$

$z_{13} - c_{13} = u_1 + v_3 - c_{13} = 4 + 1 - 6 = 5 - 6 = -1$

$z_{21} - c_{21} = u_2 + v_1 - c_{21} = 2 - 1 - 2 = -1$

$z_{22} - c_{22} = u_2 + v_2 - c_{22} = 2 - 2 - 4 = 2 - 6 = -4$

$z_{31} - c_{31} = u_3 + v_1 - c_{31} = 5 - 1 - 4 = 5 - 5 = 0$

$z_{33} - c_{33} = u_3 + v_3 - c_{33} = 5 + 1 - 8 = 6 - 8 = -2$

Step-5

Since all $z_{ij} - c_{ij} \leq 0$

the current basic feasible solution is an optimum one.

	D_1	D_2	D_3	D_4	u_i
S_1	$\boxed{3}$ 3	(-5) 7	(-1) 6	$\boxed{2}$ 4	4
S_2	(-1) 2	(-4) 4	$\boxed{2}$ 3	$\boxed{\varepsilon_1}$ 2	2
S_3	(0) 4	$\boxed{3}$ 3	(-2) 8	$\boxed{\varepsilon_2}$ 5	5
v_j	-1	-2	1	0	

The optimum Solution is

$x_{11} = 3$, $x_{14} = 2$, $x_{23} = 2$, $x_{24} = \varepsilon_1$

$x_{32} = 3$, $x_{34} = \varepsilon_2$

The transporation cost associated with the optimum schedule is.

$$3\times3+2\times4+2\times3+2\times\varepsilon_1+3\times3+5\times\varepsilon_2$$

$$9+8+6+2\varepsilon_1+9+5\varepsilon_2$$

273

$$32+2\varepsilon_1+5\varepsilon_2$$

1032. Solve the following transporation problem.

To.

	9	12	9	6	9	10	5
from.	7	3	7	7	5	5	6
	6	5	9	11	3	11	2
	6	8	11	2	2	10	9
	4	4	6	2	4	2	22

Solu:-

C12

C11	9	12	9 $\boxed{5}$	6	9	10	5 u_1 (3)(3)(0)(0)
C21	7	3 $\boxed{4}$	7	7	5	$\boxed{2}$ 5	6 - 2 = 4 u_2 (2)(2)(2)(4
C3X	$\boxed{1}$ 6	5	$\boxed{1}$ 9	11	3	11	2-1 = 1 u_3 (2)(2)(2)(
C41	$\boxed{3}$ 6	8	11	$\boxed{2}$ 2	$\boxed{4}$ 2	$\boxed{\varepsilon_1}$ 10	9-2=7-4=3 u_4 (0)(0)(4)
v_1	4-3 =1 (0)	4 (2) v_2	6-1 v_3 =5 (2)	2 (4) v_4	4 (1) v_5	2 (5) v_6	
	(0)	(2)	(2)	(4)	(1)	—	
	(0)	(2)	(2)	←	(1)	—	
	(0)	(2)	(2)				

The equation $u_i+v_j=c_{ij}$ assume that

$u_4=0$

~~step-5~~

$u_4+v_1=c_{41}$

$0+v_1=6$

$\boxed{v_1=6}$

$u_4+v_2=c_{42}$

$0+v_2=8$

$\boxed{v_2=8}$

×

$u_4+v_3=c_{43}$

$0+v_3=11$

$\boxed{v_3=11}$

×

$u_4+v_4=c_{44}$

$0+v_4=2$

$\boxed{v_4=2}$

$u_4+v_5=c_{45}$

$0+v_5=2$

$\boxed{v_5=2}$

$u_4+v_6=c_{46}$

$0+v_6=10$

$\boxed{v_6=10}$

step-3

$u_3+v_1=c_{31}$

$u_3+6=6$

$\boxed{u_3=0}$

$u_3+v_3=c_{33}$

$0+v_3=9$

$\boxed{v_3=9}$

$u_1+v_3=c_{13}$

$u_1+9=9$

$\boxed{u_1=0}$

$u_2+v_6=c_{26}$

$u_2+10=5$

$u_2=5-10$

$\boxed{u_2=-5}$

$u_2+v_2=c_{22}$

$-5+v_2=3$

$v_2=3+5$

$\boxed{v_2=8}$

Assignment Problen

"O.R. produces an integrated solution for the good of entire organization and not just to effect local improvement"

11·1. INTRODUCTION

The assignment problem is a special case of the transportation problem in which the objective is assign a number of resources to the equal number of activities at a minimum cost (or maximu profit).

Assignment problem is completely degenerate form of a transportation problem. The un available at each origin (resource) and units demanded at each destination (activity) are all equal one. That means exactly one occupied cell in each row and each column of the transportation tab i.e., only n occupied (basic) cells in place of the required $n + n - 1$ $(= 2n - 1)$.

11·2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a problem of assignment of n resources (workers) to n activities (jobs) so as to minimize overall cost or time in such a way that each resource can associate with one and only one job. T cost (or effectiveness) matrix (c_{ij}) is given as under :

$$\begin{array}{llcccccc} & & & \text{Activity} & & & & \text{Available} \\ & & A_1 & A_2 & \cdots & A_n & & \\ & R_1 & c_{11} & c_{12} & \cdots & c_{1n} & & 1 \\ & R_2 & c_{21} & c_{22} & \cdots & c_{2n} & & 1 \\ \text{Resource} & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ & R_n & c_{n1} & c_{n2} & \cdots & c_{nn} & & 1 \\ & \textit{Required} & 1 & 1 & \cdots & 1 & & \end{array}$$

The cost matrix is same as that of a transportation problem except that availability at each of resources and the requirement at each of the destinations is unity (due to the fact that assignments made on a one-to-one basis).

Let x_{ij} denote the assignment of ith resource to jth activity, such that

$$x_{ij} = \begin{cases} 1, & \text{if resource } i \text{ is assigned to activity } j \\ 0, & \text{otherwise} \end{cases}$$

Then, the mathematical formulation of the assignment problem is

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \text{ subject to the constraints :}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{and} \quad \sum_{j=1}^{n} x_{ij} = 1, \quad x_{ij} = 0 \text{ or } 1$$

$$\text{for all } i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, n$$

Note. In the above statement c_{ij} is the cost associated with assigning ith resource to jth activity.

Illustration. *Given below is an assignment problem, write it as a transportation problem.*

	A_1	A_2	A_3
R_1	1	2	3
R_2	4	5	1
R_3	2	1	4

Answer. Let x_{ij} denote the assignment of R_i $(i = 1, 2, 3)$ to A_j $(j = 1, 2, 3)$, such that

$$x_{ij} = \begin{cases} 1, & \text{if } R_i \text{ is assigned to } A_j \\ 0, & \text{otherwise} \end{cases}$$

Then the transportation problem is :

$$\text{Minimize } z = 1x_{11} + 2x_{12} + 3x_{13} + 4x_{21} + 5x_{22} + x_{23} + 2x_{31} + x_{32} + 4x_{33}$$

subject to the constraints :

$$\left.\begin{aligned} x_{11} + x_{12} + x_{13} &= 1 \\ x_{21} + x_{22} + x_{23} &= 1 \\ x_{31} + x_{32} + x_{33} &= 1 \end{aligned}\right\}, \quad \left.\begin{aligned} x_{11} + x_{21} + x_{31} &= 1 \\ x_{12} + x_{22} + x_{32} &= 1 \\ x_{13} + x_{23} + x_{33} &= 1 \end{aligned}\right\}.$$

$$x_{ij} = 0 \text{ or } 1 \quad \text{for} \quad i = 1, 2, 3 \quad \text{and} \quad j = 1, 2, 3.$$

***Theorem 11-1* (Reduction Theorem).** *In an assignment problem, if we add or subtr constant to every element of any row (or column) of the cost matrix $[c_{ij}]$, then an assignmen minimizes the total cost on one matrix also minimizes the total cost on the other matrix. In words, if $x_{ij} = x_{ij}^*$ minimizes*

$$z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \quad \text{with} \quad \sum_{i=1}^{n} x_{ij} = 1, \; \sum_{j=1}^{n} x_{ij} = 1 \; ; \; x_{ij} = 0 \text{ or } 1$$

then x_{ij}^* also minimizes $z^* = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^* x_{ij}$, *where* $c_{ij}^* = c_{ij} - u_i - v_j$ *for all* $i, j = 1, 2, \ldots$ *u_i, v_j are some real numbers.*

Proof. We write

$$\begin{aligned} z^* = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^* x_{ij} &= \sum_{i=1}^{n} \sum_{j=1}^{n} (c_{ij} - u_i - v_j) x_{ij} \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} - \sum_{i=1}^{n} u_i \sum_{j=1}^{n} x_{ij} - \sum_{i=1}^{n} x_{ij} \sum_{j=1}^{n} v_j \\ &= z - \sum_{i=1}^{n} u_i - \sum_{j=1}^{n} v_j, \end{aligned}$$

since
$$\sum_{i=1}^{n} x_{ij} = \sum_{j=1}^{n} x_{ij} = 1$$

This shows that the minimization of the new objective function z^* yields the same solution minimization of original objective function z because $\Sigma\, u_i$ and $\Sigma\, v_j$ are independent of x_{ij}.

Then, the mathematical formulation of the assignment problem is

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \text{ subject to the constraints :}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{and} \quad \sum_{j=1}^{n} x_{ij} = 1 \; ; \; x_{ij} = 0 \text{ or } 1$$

$$\text{for all } i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, n$$

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Illustration. *Given below is an assignment problem, write it as a transportation problem*

	A_1	A_2	A_3
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Then the transportation problem is :

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then x_{ij}^* also minimizes $z' = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}' x_{ij}$, where $c_{ij}' = c_{ij} - u_i - v_j$ for all $i, j = 1, 2, \ldots$ u_i, v_j are some real numbers.

Proof. We write

$$z' = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}' x_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} (c_{ij} - u_i - v_j) x_{ij}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} - \sum_{i=1}^{n} u_i \sum_{j=1}^{n} x_{ij} - \sum_{i=1}^{n} x_{ij} \sum_{j=1}^{n} v_j$$

$$= z - \sum_{i=1}^{n} u_i - \sum_{j=1}^{n} v_j ,$$

since
$$\sum_{i=1}^{n} x_{ij} = \sum_{j=1}^{n} x_{ij} = 1$$

This shows that the minimization of the new objective function z' yields the same solution minimization of original objective function z because Σu_i and Σv_j are independent of x_{ij}.

Theorem 11·2 *If $c_{ij} \geq 0$ such that minimum $\sum_{i}\sum_{j} c_{ij} = 0$, then x_{ij} provides an optimum assignment.*

The proof is left as an exercise to the reader.

The above two theorems form the basis of *Assignment Algorithm*. By selecting suitable constants to be added to or subtracted from the elements of the cost matrix we can ensure that each $c_{ij}' \geq 0$ and can produce at least one $c_{ij}' = 0$ in each row and each column and try to make assignments from among these 0 positions. The assignment schedule will be optimal if there is exactly one assignment i.e., exactly one assigned 0s in each row and each column.

Remarks. It may be noted that assignment problem is a variation of transportation problem with two characteristics (i) the cost matrix is a square matrix, and (ii) the optimum solution for the problem would always be such that there would be only one assignment in a given row or column of the cost matrix.

11·3. SOLUTION METHODS OF ASSIGNMENT PROBLEM

An assignment problem can be solved using the following four methods :

1. Complete Enumeration Method. In this method, a list of all possible assignments among the given resources and activities is prepared. Then an assignment involving the minimum cost, time or distance (or maximum profit) is selected. It represents the optimum solution. In case there are more than one assignment patterns involving the same least cost, then they all represent the optimum solutions – the problem has multiple optima.

In general, if there are n jobs and n workers, there are n ! possible assignments. Thus, the listing and evaluation of all the possible assignments is a simple matter when n is small. When n is large, the method is not very practical. For example, if there are 8 jobs and 8 workers, we have to evaluate a total of 8 ! or 40,320 assignments. The method, therefore, is not suitable for real world situations.

2. Transportation Method. Since an assignment problem is a special case of the transportation problem, it can be solved by transportation methods discussed in the previous chapter. However, every basic feasible solution of a general assignment problem having a square payoff matrix of order n should have $m + n - 1 = n + n - 1 = 2n - 1$ assignments or basic cells. But due to the special structure of this problem any basic solution cannot have more than n assignments. Thus, the assignment problem is inherently degenerate. In order to remove degeneracy, $(n - 1)$ dummy allocations will be required to proceed with the transportation method. However, because of the large number of dummy allocations in the solution, the transportation method becomes computationally inefficient for solving an assignment problem.

3. Simplex Method. An assignment problem can be formulated as a transportation problem which, in turn, is itself a special case of an LPP. Accordingly, an assignment problem can be formulated as an LPP with integer valued variables and may be solved using a modified simplex method or otherwise. Here, the decision variables take only one of the two values : 1 or 0.

In general let

$$x_{ij} = \begin{cases} 1 & \text{if } i\text{th person is assigned } j\text{th job} \\ 0 & \text{if } i\text{th person is not assigned } j\text{th job} \end{cases}$$

The mathematical formulation of the assignment problem as a 0-1 integer linear programming problem would be :

$$\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \text{ subject to the constraints :}$$

$$x_{i1} + x_{i2} + \dots + x_{in} = 1 ; \qquad i = 1, 2, \dots, n$$

$$x_{1j} + x_{2j} + \dots + x_{nj} = 1 ; \qquad j = 1, 2, \dots, n$$

$$x_{ij} = 0 \text{ or } 1 \text{ for all } i \text{ and } j.$$

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As can be seen in the general mathematical formulation of the assignment proble $n \times n$ decision variables and $n + n$ or $2n$ equalities/equations. In particular, for a problem workers/jobs, there will be 25 decision variables and 10 equalities. That means a simplex 25 columns and 10 rows. It is difficult to solve manually and hence this approach to th not considered

4. Hungarian Assignment Method. An efficient method for solving an assignmen the *Hungarian Assignment Method* (also known as reduced matrix method) which is concept of opportunity cost. Opportunity costs show the relative penalties associated wit of resource to an activity as opposed to making the best or least cost assignment. If we ca cost matrix to the extent of having at least one zero in each row and each column, th possible to make optimal assignments (opportunity costs are all zero).

The method of solving an assignment problem (minimization case) consists of th steps :

Step 1. Determine the cost table from the given problem.

(*i*) If the number of sources is equal to the number of destinations, go to *step 3.*

(*ii*) If the number of sources is not equal to the number of destinations, go to *step 2*

Step 2. Add a dummy source or dummy destination, so that the cost table becom matrix. The cost entries of dummy source/destinations are always zero.

Step 3. Locate the smallest element in each row of the given cost matrix and then same from each element of that row.

Step 4. In the reduced matrix obtained in *step 3,* locate the smallest element of each then subtract the same from each element of that column. Each column and row now have zero.

Step 5. In the modified matrix obtained in *step 4,* search for an optimal assignment

(*a*) Examine the rows successively until a row with a single zero is found. Enrectan (☐) and cross off (×) all other zeros in its column. Continue in this manner until have been taken care of.

(*b*) Repeat the procedure for each column of the reduced matrix.

(*c*) If a row and/or column has two or more zeros and one cannot be chosen by insp assign arbitrary any one of these zeros and cross off all other zeros of that row/c

(*d*) Repeat (*a*) through (*c*) above successively until the chain of assigning (☐) or cro

Step 6. If the number of assignments (☐) is equal to *n* (the order of the cost matrix), solution is reached.

If the number of assignments is less than *n* (the order of the matrix), go to the next

Step 7. Draw the minimum number of horizontal and/or vertical lines to cover all the reduced matrix. This can be conveniently done by using a simple procedure :

(*a*) Mark (√) rows that do not have any assigned zero.

(*b*) Mark (√) columns that have zeros in the marked rows.

(*c*) Mark (√) rows that have assigned zeros in the marked columns.

(*d*) Repeat (*b*) and (*c*) above until the chain of marking is completed.

(*e*) Draw lines through all the *unmarked rows* and *marked columns.* This gives us minimum number of lines.

Step 8. Develop the new revised cost matrix as follows :

(*a*) Find the smallest element of the reduced matrix not covered by any of the lines.

(*b*) Subtract this element from all the *uncovered* elements and add the same to all t lying at the *intersection* of any two lines.

Step 9. Go to *Step 6* and repeat the procedure until an optimum solution is attained.

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SAMPLE PROBLEMS

1101. *A departmental head has four subordinates, and four tasks to be performed. Th bordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate, of th me each man would take to perform each task, is given in the matrix below*

Tasks	*Men*			
	E	*F*	*G*	*H*
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a man, so as to minimize the total man-hours?
[Andhra B.E. (Mech. & Ind.) 19

Solution.

Step 1. Here, the number of tasks and the number of subordinates each equal 4. therefore problem is balanced and we move on to *step 3.*

Step 3. Subtracting the smallest element of each row from every element of the correspond row, we get the reduced matrix

7	15	6	0
0	15	1	13
23	4	3	0
9	16	14	0

Step 4. Subtracting the smallest element of each column of the reduced matrix from e element of the corresponding column, we get the following reduced matrix.

7	11	5	0
0	11	0	13
23	0	2	0
9	12	13	0

Step 5. Starting with row *1*, we enrectangle (□) (i.e., make assignment) a single zero, if an cross (×) all other zeros in the column so marked. Thus, we get

7	11	5	$\boxed{0}$
$\boxed{0}$	11	$\cancel{0}$	13
23	$\boxed{0}$	2	$\cancel{0}$
9	12	13	$\cancel{0}$

In the above matrix, we arbitrarily enrectangled a zero in column *1*, because row 2 had two

It may be noted that column *3* and row 4 do not have any assignment. So, we move on next step.

Step 7. (i) Since row 4 does not have any assignment, we mark this row (√).

(ii) Now there is a zero in the fourth column of the marked row. So, we mark fourth colum

(iii) Further there is an assignment in the first row of the marked column. So we mark first

Thus, we have

(iv) Draw straight lines through all unmarked rows and marked columns.

SAMPLE PROBLEMS

1101. *A departmental head has four subordinates, and four tasks to be performed. Th ubordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate, of th ne each man would take to perform each task, is given in the matrix below:*

Tasks	Men			
	E	*F*	*G*	*H*
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a man, so as to minimize the total man-hours?
[Andhra B.E. (Mech. & Ind.) 19

Solution.

Step 2. Here, the number of tasks and the number of subordinates each equal 4. therefore problem is balanced and we move on to *step 3.*

Step 3. Subtracting the smallest element of each row from every element of the correspond ow, we get the reduced matrix

$$\begin{array}{|cccc|} 7 & 15 & 6 & 0 \\ 0 & 15 & 1 & 13 \\ 23 & 4 & 3 & 0 \\ 9 & 16 & 14 & 0 \end{array}$$

Step 4. Subtracting the smallest element of each column of the reduced matrix from e lement of the corresponding column, we get the following reduced matrix :

$$\begin{array}{|cccc|} 7 & 11 & 5 & 0 \\ 0 & 11 & 0 & 13 \\ 23 & 0 & 2 & 0 \\ 9 & 12 & 13 & 0 \end{array}$$

Step 5. Starting with row *1*, we enrectangle (☐) (i.e., make assignment) a single zero, if an ross (×) all other zeros in the column so marked. Thus, we get

$$\begin{array}{|cccc|} 7 & 11 & 5 & \boxed{0} \\ \boxed{0} & 11 & \cancel{0} & 13 \\ 23 & \boxed{0} & 2 & \cancel{0} \\ 9 & 12 & 13 & \cancel{0} \end{array}$$

In the *above* matrix, we arbitrarily enrectangled a zero in column *1*, because row 2 had two
It may be noted that column *3* and row 4 do not have any assignment. So, we move on
ext step.

we mark this row (✓).

Step 7. (i) Since row 4 does not have any assignment, we mark fourth colum
(ii) Now there is a zero in the fourth column of the marked row. So, we mark first
(iii) Further there is an assignment in the first row of the marked column. Thus, we have
(iv) Draw straight lines through all unmarked rows and marked columns.

7	11	5	$\boxed{0}$	✓
$\boxed{0}$	11	$\cancel{0}$	13	
23	$\boxed{0}$	2	$\cancel{0}$	
9	12	13	$\cancel{0}$	✓
			✓	

Step 8. In *step 7*, we observe that the minimum number of lines so drawn is 3, which is less he order of the cost matrix, indicating that the current assignment is not optimum.

To increase the minimum number of lines, we generate new zeros in the modified matrix.

The smallest element not covered by the lines is 5. Subtracting this element from all uncovered elements and adding the same to all the elements lying at the intersection of the lines btain the following new reduced cost matrix :

2	6	0	0
0	11	0	18
23	0	2	5
4	7	8	0

Step 9. Repeating step 5 on the reduced matrix, we get

2	6	$\boxed{0}$	$\cancel{0}$
$\boxed{0}$	11	$\cancel{0}$	18
23	$\boxed{0}$	2	5
4	7	8	$\boxed{0}$

Now, since each row and each column has one and only one assignment, an optimal solution reached. The optimum assignment is :

$$A \to G,\ B \to E,\ C \to F \text{ and } D \to H.$$

The minimum total time for this assignment scheduled is 17 + 13 + 19 + 10 or 59 man-hours.

1102. *A company wishes to assign 3 jobs to 3 machines in such a way that each job is assigne to some machine and no machine works on more than one job. The cost of assigning job i to machin j is given by the matrix below (ijth entry) :*

$$\textit{Cost matrix} \quad \begin{bmatrix} 8 & 7 & 5 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{bmatrix}$$

Draw the associated network. Formulate the network LPP and find the minimum cost of making the assignment.

Solution. (a) Network formulation of the given problem is given as under :

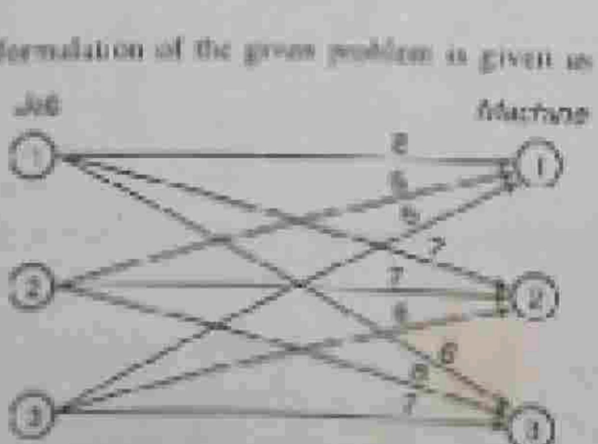

Fig. 11.1

$$\begin{array}{|cccc|l}
7 & 11 & 5 & \boxed{0} & \checkmark \\
\boxed{0} & 11 & \cancel{0} & 13 & \\
23 & \boxed{0} & 2 & \cancel{0} & \\
9 & 12 & 13 & \cancel{0} & \checkmark \\
 & & & \checkmark &
\end{array}$$

Step 8. In *step 7*, we observe that the minimum number of lines so drawn is 3, which is less
he order of the cost matrix, indicating that the current assignment is not optimum.

To increase the minimum number of lines, we generate new zeros in the modified matrix.

The smallest element not covered by the lines is 5. Subtracting this element from all uncovered elements and adding the same to all the elements lying at the intersection of the lines obtain the following new reduced cost matrix :

2	6	0	0
0	11	0	18
23	0	2	5
4	7	8	0

Step 9. Repeating step 5 on the reduced matrix, we get

$$\begin{array}{|cccc|}
2 & 6 & \boxed{0} & \cancel{0} \\
\boxed{0} & 11 & \cancel{0} & 18 \\
23 & \boxed{0} & 2 & 5 \\
4 & 7 & 8 & \boxed{0}
\end{array}$$

Now, since each row and each column has one and only one assignment, an optimal solution reached. The optimum assignment is :

A → G, B → E, C → F and D → H.

The minimum total time for this assignment scheduled is 17 + 13 + 19 + 10 or 59 man-hours.

1102. *A company wishes to assign 3 jobs to 3 machines in such a way that each job is assigne to some machine and no machine works on more than one job. The cost of assigning job i to machin j is given by the matrix below (ijth entry) :*

$$\textit{Cost matrix} \quad \begin{bmatrix} 8 & 7 & 5 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{bmatrix}$$

Draw the associated network. Formulate the network LPP and find the minimum cost of making the assignment.

Solution. (a) Network formulation of the given problem is given as under :

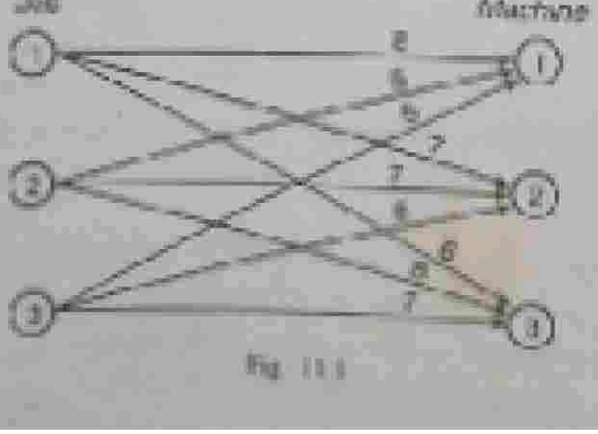

Fig. 11.1

) Linear programming formulation of the given problem is
inimize the total cost involved, *i.e.*,

Minimize $z = (8x_{11} + 7x_{12} + 6x_{13}) + (5x_{21} + 7x_{22} + 8x_{23}) + (6x_{31} + 8x_{32} + 7x_{33})$

subject to the constraints ,

$$x_{i1} + x_{i2} + x_{i3} = 1, \qquad i =$$
$$x_{1j} + x_{2j} + x_{3j} = 1, \qquad j =$$
$$x_{ij} = 0 \text{ or } 1, \text{ for all } i \text{ and } j$$

) Reduce the cost matrix by subtracting smallest element of each row (column) fr ceeding row (column) elements. In the reduced matrix, make assignments in rows and co ng single zeros and cross off all other zeros in rows and columns in the which the assignme made. See table 11.1. Now, draw the minimum number of lines to cover all the zeros. F oceed as follows:

(i) Mark (√) third row since it has no assignment.
(ii) Mark (√) first column, since third row has a zero in this column.
(iii) Mark (√) second row, since marked column has an assignment in the second row.
(iv) Since no other row or column can be marked, draw straight lines through the unmarke and marked column as shown in table 11.1 :

2	0	$\boxed{0}$	
$\boxed{0}$	1	3	√
$\cancel{0}$	1	1	√
√			

Table 11.1

3	$\boxed{0}$	$\cancel{0}$
$\boxed{0}$	$\cancel{0}$	2
$\cancel{0}$	$\cancel{0}$	$\boxed{0}$

Table 11.2

Modify the reduced cost matrix (table 11.1) by selecting the smallest element among covered elements. Subtract this element from all the uncovered elements including itself and the intersection element (1, 1) which lies at the intersection of two lines. The modified cost o obtained is shown in table 11.2.

In table 11.2, we observe that there is no row and column which has single zero. So, we m ssignment arbitrarily at (1, 2) and cross off all zeros of first row and second column. Now, w ngle zero in the second row and therefore an assignment is made at (2, 1). Cross off all zeros first column. Finally, we make an assignment at (3, 3) being the single zero in the third row.

Clearly, the number of assignments in table 11.2 is equal to the order of the matrix. He optimum assignment has been attained, *viz.*,

Job 1 → Machine 2, Job 2 → Machine 1, Job 3 → Machine 3.

Total minimum cost will be (7 + 5 + 7), *i.e.*, 19.

1103. *A pharmaceutical company is producing a single product and is selling it throu agencies located in different cities. All of a sudden, there is a demand for the product in anot cities not having any agency of the company. The company is faced with the problem of decid how to assign the existing agencies to despatch the product to needy cities in such a way t travelling distance is minimized. The distance between the surplus and deficit cities (in km) i in the following table :*

		Deficit cities				
		a	*b*	*c*	*d*	*e*
Surplus cities	*A*	85	75	65	125	75
	B	90	78	66	132	78
	C	75	66	57	114	69
	D	80	72	60	120	72
	E	76	64	56	112	68

Determine the optimum assignment schedule. [Delhi M.B.A. (HCA) PT. No

)) Linear programming formulation of the given problem is

inimize the total cost involved, *i.e.*,

Minimize $z = (8x_{11} + 7x_{12} + 6x_{13}) + (5x_{21} + 7x_{22} + 8x_{23}) + (6x_{31} + 8x_{32} + 7x_{33})$

subject to the constraints ,

$$x_{i1} + x_{i2} + x_{i3} = 1, \qquad i =$$

$$x_{1j} + x_{2j} + x_{3j} = 1, \qquad j =$$

$$x_{ij} = 0 \text{ or } 1, \text{ for all } i \text{ and } j$$

c) Reduce the cost matrix by subtracting smallest element of each row (column) fr ucceeding row (column) elements. In the reduced matrix, make assignments in rows and co ng single zeros and cross off all other zeros in rows and columns in the which the assignme made. See table *11.1*. Now, draw the minimum number of lines to cover all the zeros. F roceed as follows:

(i) Mark (√) third row since it has no assignment.

(ii) Mark (√) first column, since third row has a zero in this column.

(iii) Mark (√) second row, since marked column has an assignment in the second row.

(iv) Since no other row or column can be marked, draw straight lines through the unmarke and marked column as shown in table *11.1* :

2	0	$\boxed{0}$	
$\boxed{0}$	1	3	√
$\cancel{0}$	1	1	√
√			

Table 11.1

3	$\boxed{0}$	$\cancel{0}$
$\boxed{0}$	$\cancel{0}$	2
$\cancel{0}$	$\cancel{0}$	$\boxed{0}$

Table 11.2

Modify the reduced cost matrix (table *11.1*) by selecting the smallest element among covered elements. Subtract this element from all the uncovered elements including itself and the intersection element (1, 1) which lies at the intersection of two lines. The modified cost obtained is shown in table *11.2*.

In table *11.2*, we observe that there is no row and column which has single zero. So, we m ignment arbitrarily at (1, 2) and cross off all zeros of first row and second column. Now, w ngle zero in the second row and therefore an assignment is made at (2, 1). Cross off all zeros rst column. Finally, we make an assignment at (3, 3) being the single zero in the third row.

Clearly, the number of assignments in table *11.2* is equal to the order of the matrix. He ptimum assignment has been attained, *viz.*,

Job *1* → Machine 2, Job *2* → Machine *1*, Job *3* → Machine *3*.

Total minimum cost will be (7 + 5 + 7), *i.e.*, 19.

1103. *A pharmaceutical company is producing a single product and is selling it throu agencies located in different cities. All of a sudden, there is a demand for the product in anoth cities not having any agency of the company. The company is faced with the problem of decid how to assign the existing agencies to despatch the product to needy cities in such a way t avelling distance is minimised. The distance between the surplus and deficit cities (in km) i in the following table :*

		Deficit cities				
		a	*b*	*c*	*d*	*e*
Surplus cities	*A*	85	75	65	125	75
	B	90	78	66	132	78
	C	75	66	57	114	69
	D	80	72	60	120	72
	E	76	64	56	112	68

Determine the optimum assignment schedule. [Delhi M.B.A. (HCA) PT. No

Solution. Subtracting the smallest element of each row from every element of that and th btracting the smallest element of each column from every element of that column, we get d duced distance table :

	a	b	c	d	e
A	2	2	0	4	0
B	6	4	0	10	2
C	0	1	0	1	2
D	2	4	0	4	2
E	2	0	0	0	2

Table 11.3

In the reduced distance table, we make assignments in rows and columns having single zeros a ross off all other zeros in those rows and columns, where assignments have been made. Now dra e minimum number of lines to cover all the zeros. This is done in the following steps :

(i) Mark (√) row '*D*' since it has no assignment.

(*ii*) Mark (√) column '*C*' since row '*D*' has zero in this column.

(*iii*) mark (√) row *B* since column '*C*' has an assignment in row '*B*'.

(*iv*) Since no other rows or columns can be marked, draw straight lines through the unmark rows '*A*' '*C*', and '*E*', and marked column '*C*' as shown in Table *11.4*.

	a	b	c	d	e	
A	2	2	$\cancel{0}$	4	$\boxed{0}$	
B	6	4	$\boxed{0}$	10	2	√
C	$\boxed{0}$	1	$\cancel{0}$	1	2	
D	2	4	$\cancel{0}$	4	2	√
E	2	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	2	
			√			

Table 11.4

	a	b	c	d	e
A	2	2	2	4	0
B	4	2	0	8	0
C	0	1	2	1	2
D	0	2	0	2	0
E	2	0	2	0	2

Table 11.5

Modify the reduced distance table (Table *11.4*) by subtracting the smallest element not covered b lines from all the uncovered elements and add the same at the intersection elements of the lines. T modified distance table so obtain is shown in Table *11.5*.

Repeat the above procedure to find the new assignment in table *11.6*

	a	b	c	d	e	
A	2	2	2	4	$\boxed{0}$	√
B	4	2	$\boxed{0}$	8	$\cancel{0}$	√
C	$\boxed{0}$	1	2	1	2	√
D	$\cancel{0}$	2	$\cancel{0}$	2	$\cancel{0}$	√
E	2	$\boxed{0}$	2	$\cancel{0}$	2	
	√		√		√	

Table 11.6

	a	b	c	d	e
A	2	1	2	3	$\boxed{0}$
B	4	1	$\boxed{0}$	7	$\cancel{0}$
C	$\cancel{0}$	$\boxed{0}$	2	$\cancel{0}$	2
D	$\boxed{0}$	1	$\cancel{0}$	1	$\cancel{0}$
E	3	$\cancel{0}$	3	$\boxed{0}$	3

Table 11.7

Clearly the assignment shown in table *11.6* is also not optimum, since only four assignments a made. To get the next solution, we draw the minimum number of horizontal and vertical lines to co all the zeros in table *11.6*. Subtracting the smallest uncovered element (viz., 1) from all uncover elements and adding the same to the intersection element of two lines gives us table *11.7*.

Solution. Subtracting the smallest element of each row from every element of that and the btracting the smallest element of each column from every element of that column, we get th duced distance table :

	a	b	c	d	e
A	2	2	0	4	0
B	6	4	0	10	2
C	0	1	0	1	2
D	2	4	0	4	2
E	2	0	0	0	2

Table 11.3

In the reduced distance table, we make assignments in rows and coloumns having single zeros a ross off all other zeros in those rows and columns, where assignments have been made. Now dra he minimum number of lines to cover all the zeros. This is done in the following steps :

(i) Mark (√) row '*D*' since it has no assignment.

(*ii*) Mark (√) coloumn '*C*' since row '*D*' has zero in this column.

(*iii*) mark (√) row *B* since column '*C*' has an assignment in row '*B*'.

(*iv*) Since no other rows or columns can be marked, draw straight lines through the unmark rows '*A*' '*C*', and '*E*', and marked coloumn '*C*' as shown in Table *11.4*.

	a	b	c	d	e	
A	2	2	$\cancel{0}$	4	$\boxed{0}$	
B	6	4	$\boxed{0}$	10	2	√
C	$\boxed{0}$	1	$\cancel{0}$	1	2	
D	2	4	$\cancel{0}$	4	2	√
E	2	$\boxed{0}$	$\cancel{0}$	0	2	
			√			

Table 11.4

	a	b	c	d	e
A	2	2	2	4	0
B	4	2	0	8	0
C	0	1	2	1	2
D	0	2	0	2	0
E	2	0	2	0	2

Table 11.5

Modify the reduced distance table (Table *11.4*) by subtracting the smallest element not covered b lines from all the uncovered elements and add the same at the intersection elements of the lines. Th modified distance table so obtain is shown in Table *11.5*.

Repeat the above procedure to find the new assignment in table *11.6*

	a	b	c	d	e	
A	2	2	2	4	$\boxed{0}$	√
B	4	2	$\boxed{0}$	8	$\cancel{0}$	√
C	$\boxed{0}$	1	2	1	2	√
D	$\cancel{0}$	2	$\cancel{0}$	2	$\cancel{0}$	√
E	2	$\boxed{0}$	2	0	2	
	√		√		√	

Table 11.6

	a	b	c	d	e
A	2	1	2	3	$\boxed{0}$
B	4	1	$\boxed{0}$	7	$\cancel{0}$
C	$\cancel{0}$	$\boxed{0}$	2	$\cancel{0}$	2
D	$\boxed{0}$	1	$\cancel{0}$	1	$\cancel{0}$
E	3	$\cancel{0}$	3	$\boxed{0}$	3

Table 11.7

Clearly the assignment shown in table *11.6* is also not optimum, since only four assignments a made. To get the next solution, we draw the minimum number of horizontal and vertical lines to cov all the zeros in table *11.6*. Subtracting the smallest uncovered element (viz., 1) from all uncover elements and adding the same to the intersection element of two lines gives us table *11.7*.

The new assignment schedule is shown in table 11.7. Since, both the rows '*C*' and ' zeros, the arbitrary selection of a cell in any of these two rows will give us an alternat having the same total distance.

Since the number of assignments is equal to the order of the given matrix, an optim is attained, viz.,

A → *e*, *B* → *c*, *C* → *b*, *D* → *a*, *E* → *d*, or *A* → *e*, *B* → *c*, *C* → *d*, *D* → *a*,

Total minimum distance in both the cases will be 399 kilometres.

1104. *A department head has four tasks to be performed and three subordinates, the differ in efficiency. The estimates of the time, each subordinate would take to perform, is in the matrix. How should he allocate the tasks one to each man, so as to minimi man-hours?*

Task	Men		
	1	*2*	*3*
I	9	26	1
II	13	27	
III	35	20	1
IV	18	30	2

Solution. Here we have three subordinates who have to perform four tasks. So, the g is unbalanced and therefore we add a dummy subordinate (column) with all its entries resulting balanced problem is :

Subordinate		*1*	*2*	*3*
Task	*I*	9	26	15
	II	15	27	6
	III	35	30	15
	IV	18	30	20

Now, reduce the balanced time-matrix by subtracting the smallest element of each all the elements of that column. In the reduced matrix, make assignments in rows and col single zeros and cross off all other zeros of the rows and columns, where assignment hav We get the following assignment solution :

$$\begin{array}{c|cccc|}
 & 1 & 2 & 3 & \text{Dummy} \\
\hline
I & \boxed{0} & 6 & 9 & \boxtimes \\
II & 4 & 7 & \boxed{0} & \boxtimes \\
III & 26 & \boxed{0} & 9 & \boxtimes \\
IV & 9 & 10 & 14 & \boxed{0} \\
\hline
\end{array}$$

Table 11.8

The optimum assignment is

I → *1*, *II* → *3* and *III* → *2* ; while task *IV* should be assigned to a dummy man, i to be done. The minimum time is 35 hours.

PROBLEMS

1105. Four professors are each capable of teaching any one of four different courses. Class p in hours for different topics varies from professor to professor and is given in the table below. E assigned only one course. Determine an assignment schedule so as to minimize the total course p for all courses.

The new assignment schedule is shown in table 11.7. Since, both the rows '*c*' and ' zeros, the arbitrary selection of a cell in any of these two rows will give us an alterna having the same total distance.

Since the number of assignments is equal to the order of the given matrix, an optim is attained, viz.,

$A \to e,\ B \to c,\ C \to b,\ D \to a,\ E \to d,$ or $A \to e,\ B \to c,\ C \to d,\ D \to a,$

Total minimum distance in both the cases will be 399 kilometres.

1104. *A department head has four tasks to be performed and three subordinates, the differ in efficiency. The estimates of the time, each subordinate would take to perform, is in the matrix. How should he allocate the tasks one to each man, so as to minimi man-hours?*

Task	*Men*		
	1	*2*	*3*
I	9	26	
II	13	27	
III	35	20	
IV	18	30	

Solution. Here we have three subordinates who have to perform four tasks. So, the gi is unbalanced and therefore we add a dummy subordinate (column) with all its entries resulting balanced problem is :

Subordinate		*1*	*2*	*3*	*4*
Task	*I*	9	26	15	
	II	13	27	6	
	III	35	20	15	
	IV	18	30	20	

Now, reduce the balanced time-matrix by subtracting the smallest element of each all the elements of that column. In the reduced matrix, make assignments in rows and col single zeros and cross off all other zeros of the rows and columns, where assignment hav We get the following assignment solution :

	1	*2*	*3*	*Dummy*
I	$\boxed{0}$	6	9	$\cancel{0}$
II	4	7	$\boxed{0}$	$\cancel{0}$
III	26	$\boxed{0}$	9	$\cancel{0}$
IV	9	10	14	$\boxed{0}$

Table 11.8

The optimum assignment is

I → *1*, *II* → *3* and *III* → *2* ; while task *IV* should be assigned to a dummy man, *i* to be done. The minimum time is 35 hours.

PROBLEMS

1105. Four professors are each capable of teaching any one of four different courses. Class p in hours for different topics varies from professor to professor and is given in the table below. Ea assigned only one course. Determine an assignment schedule so as to minimize the total course p for all courses.

11:4. SPECIAL CASES IN ASSIGNMENT PROBLEMS

Maximization Case in Assignment Problem. In some cases, the pay off elements of the assignme problem may represent revenues or profits instead of costs so that the objective will be to maxim the total revenue or profit. The Hungarian method explained earlier can also be used for maximizati case. The problem of maximization can be converted into a minimization one by selecting the larges element of the profit matrix and then subtracting from it all the elements of the matrix. This gives an opportunity loss matrix. We then proceed as usual and obtain the optimum assignment schedu Maximum revenue/profit is then obtained by restoring the original values of those cells to which assignments have been made.

Prohibited Assignments. Sometimes due to certain reasons, a particular resource (say a man or a machine) cannot be assigned to perform a particular activity (say a territory or a job). In such case the cost of performing that particular activity by a particular resource is considered to be very larg (written as *M* or ∞) so as to prohibit the entry of this pair of resource-activity into the final solutio

SAMPLE PROBLEMS

1124. *A manufacturing company has four zones A, B, C, D and four sales engineers P, Q, R, respectively for assignment. Since the zones are not equally rich in sales potential, it is estimated tha a particular engineer operating in a particular zone will bring the following sales :*

Zone A : 4,20,000, Zone B : 3,36,000, Zone C : 2,94,000, Zone D : 4,62,000

The engineers are having different sales ability. Working under the same conditions their year sales are proportional to 14, 9, 11 and 8 respectively. The criteria of maximum expected total sal is to be met by assigning the best engineer to the richest zone, the next best to the second richest zo and so on.

[C.A. Final (May) 199

Find the optimum assignment and the maximum sales.

Solution. *Step 1. Construct the Effectiveness Matrix.* To avoid the fractional values of annu sales of each sales engineer in each zone, for convenience consider their yearly sales as 42 (i.e., th sum of sales proportions), taking Rs. 1000 as one unit. Now divide the individual sales in each zon by 42 to obtain the required annual sales by each sales engineer. For example, if sales engineer *P* i assigned to zone *A*, the annual sales is 14 × 4,20,000/42, i.e., 1,40,000. Thus, the proportional sales the four zones are given in the following effective table (matrix) :

		Zones				Sales proportion
		A	*B*	*C*	*D*	
	P	140	112	98	154	14
Sales	*Q*	90	72	63	99	9
engineer	*R*	110	88	77	121	11
	S	80	64	56	88	8

Step 2. Converting Maximization problem into Minimization problem : The given maximizatio assignment problem (table) can be converted into a minimization assignment problem by subtracti from the highest element (i.e., 154), all the elements of the given table. The resulting opportunity lo matrix is :

	A	*B*	*C*	*D*
P	14	42	56	0
Q	64	82	91	55
R	44	66	77	33
S	74	90	98	66

Step 3. Optimum Assignment : Now, in the opportunity loss matrix subtract the minimu element of each row from all the elements of that row. Then subtract the minimum element of eac

11:4. SPECIAL CASES IN ASSIGNMENT PROBLEMS

Maximization Case in Assignment Problem. In some cases, the pay off elements of the assignment problem may represent revenues or profits instead of costs so that the objective will be to maximize the total revenue or profit. The Hungarian method explained earlier can also be used for maximization case. The problem of maximization can be converted into a minimization one by selecting the largest element of the profit matrix and then subtracting from it all the elements of the matrix. This gives us an opportunity loss matrix. We then proceed as usual and obtain the optimum assignment schedule. Maximum revenue/profit is then obtained by restoring the original values of those cells to which the assignments have been made.

Prohibited Assignments. Sometimes due to certain reasons, a particular resource (say a man or a machine) cannot be assigned to perform a particular activity (say a territory or a job). In such cases, the cost of performing that particular activity by a particular resource is considered to be very large (written as *M* or ∞) so as to prohibit the entry of this pair of resource-activity into the final solution.

SAMPLE PROBLEMS

1124. *A manufacturing company has four zones A, B, C, D and four sales engineers P, Q, R, S respectively for assignment. Since the zones are not equally rich in sales potential, it is estimated that a particular engineer operating in a particular zone will bring the following sales :*

Zone A : 4,20,000, Zone B : 3,36,000, Zone C : 2,94,000, Zone D : 4,62,000

The engineers are having different sales ability. Working under the same conditions their yearly sales are proportional to 14, 9, 11 and 8 respectively. The criteria of maximum expected total sales is to be met by assigning the best engineer to the richest zone, the next best to the second richest zone and so on.

Find the optimum assignment and the maximum sales. [C.A. Final (May) 199

Solution. *Step 1. Construct the Effectiveness Matrix.* To avoid the fractional values of annual sales of each sales engineer in each zone, for convenience consider their yearly sales as 42 (i.e., the sum of sales proportions), taking Rs. 1000 as one unit. Now divide the individual sales in each zone by 42 to obtain the required annual sales by each sales engineer. For example, if sales engineer P is assigned to zone A, the annual sales is 14 × 4,20,000/42, i.e., 1,40,000. Thus, the proportional sales in the four zones are given in the following effective table (matrix) :

		Zones A	B	C	D	Sales proportion
Sales engineer	P	140	112	98	154	14
	Q	90	72	63	99	9
	R	110	88	77	121	11
	S	80	64	56	88	8

Step 2. Converting Maximization problem into Minimization problem : The given maximization assignment problem (table) can be converted into a minimization assignment problem by subtracting from the highest element (i.e., 154), all the elements of the given table. The resulting opportunity loss matrix is :

	A	B	C	D
P	14	42	56	0
Q	64	82	91	55
R	44	66	77	33
S	74	90	98	66

Step 3. Optimum Assignment : Now, in the opportunity loss matrix subtract the minimum element of each row from all the elements of that row. Then subtract the minimum element of each

umn from all the elements of that column. In the reduced matrix make assignments in rows and columns that have single zeros as usual. Thus, we have

Initial Iteration. Draw the minimum number of lines to cover all the zeros of the reduced matri
See table *11.9*

First iteration. Modify the reduced opportunity loss matrix by subtracting element '1' from all the elements not covered by the lines and adding the same at the intersection of two lines. Thus, we get table *11.10*

	A	*B*	*C*	*D*	
P	6	18	24	$\boxed{0}$	√
Q	1	3	4	$\cancel{0}$	√
R	3	9	12	$\cancel{0}$	√
S	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	
				√	

Table 11.9

	A	*B*	*C*	*D*	
P	5	17	23	$\boxed{0}$	√
Q	$\boxed{0}$	2	3	$\cancel{0}$	
R	2	8	11	$\cancel{0}$	√
S	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	1	
				√	

Table 11.10

Second iteration. Modify further the reduced table *11.10* by subtracting element '2' from all the elements not covered by the lines and adding the same at the intersection of two lines. Thus, we get table *11.11*

Final iteration. Modify further the reduced table *11.11* by subtracting element '2' from all the elements not covered by the lines and adding the same at the intersection of two lines. Thus, we get table *11.12*

	A	*B*	*C*	*D*	
P	3	15	21	$\boxed{0}$	√
Q	$\boxed{0}$	2	3	2	√
R	$\cancel{0}$	6	9	$\cancel{0}$	√
S	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	3	
	√			√	

Table 11.11

	A	*B*	*C*	*D*
P	1	13	15	$\boxed{0}$
Q	$\cancel{0}$	$\boxed{0}$	1	2
R	$\boxed{0}$	4	7	$\cancel{0}$
S	2	$\cancel{0}$	$\boxed{0}$	5

Table 11.12

The optimum solution is obtained and the assignment is :

$$P \rightarrow D,\ Q \rightarrow B,\ R \rightarrow A \text{ and } S \rightarrow C$$

The solution shows that the best salesman *P* is assigned to the richest zone *D* and the wo salesman *S* to the poorest zone *C*. The second best salesman to the next richest zone *A*, and so on

Maximum sales = Rs. 154 + 77 + 110 + 56 = Rs. 292 thousands

Problem 11.25. *A student has to select one and only one elective in each semester and the sa elective should not be selected in different semesters. Due to various reasons, the expected grade in each subject if selected in different semesters vary and they are given below.*

Semester	*Analysis*	*Statistics*	*Graph Theory*	*Algebra*
I	*F*	*E*	*D*	*C*
II	*E*	*E*	*C*	*C*
III	*C*	*D*	*C*	*A*
IV	*B*	*A*	*H*	*H*

The grade points are: *H* = 10, *A* = 9, *B* = 8, *C* = 7, *D* = 6, *E* = 5 and *F* = 4. How will the student select the electives in order to maximize the total expected points and what will be the maximum expected total points? *[Delhi B.Sc. (Stat.) 2* column from all the elements of that column. In the reduced matrix make assignments in rows and columns having single zeros as usual. Thus, we have

Initial Iteration. Draw the minimum number of lines to cover all the zeros of the reduced matrix. See table 11.9

First iteration. Modify the reduced opportunity loss matrix by subtracting element '1' from all the elements not covered by the lines and adding the same at the intersection of two lines. Thus, we get table 11.10

	A	B	C	D	
P	6	18	24	$\boxed{0}$	√
Q	1	3	4	$\cancel{0}$	√
R	3	9	12	$\cancel{0}$	√
S	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	
				√	

Table 11.9

	A	B	C	D	
P	5	17	23	$\boxed{0}$	√
Q	$\boxed{0}$	2	3	$\cancel{0}$	
R	2	8	11	$\cancel{0}$	√
S	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	1	
				√	

Table 11.10

Second iteration. Modify further the reduced table 11.10 by subtracting element '2' from all the elements not covered by the lines and adding the same at the intersection of two lines. Thus, we get table 11.11

Final iteration. Modify further the reduced table 11.11 by subtracting element '2' from all the elements not covered by the lines and adding the same at the intersection of two lines. Thus, we get table 11.12

	A	B	C	D	
P	3	15	21	$\boxed{0}$	√
Q	$\boxed{0}$	2	3	2	√
R	$\cancel{0}$	6	9	$\cancel{0}$	√
S	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	3	
	√			√	

Table 11.11

	A	B	C	D
P	3	13	19	$\boxed{0}$
Q	$\cancel{0}$	$\boxed{0}$	1	2
R	$\boxed{0}$	4	7	$\cancel{0}$
S	2	$\cancel{0}$	$\boxed{0}$	5

Table 11.12

The optimum solution is obtained and the assignment is :

$$P \rightarrow D,\ Q \rightarrow B,\ R \rightarrow A \text{ and } S \rightarrow C$$

The solution shows that the best salesman P is assigned to the richest zone D and the worst salesman S to the poorest zone C. The second best salesman to the next richest zone A, and so on.

Maximum sales = Rs. 154 + 72 + 110 + 56 = Rs. 392 thousands

Problem 11.25. *A student has to select one and only one elective in each semester and the same elective should not be selected in different semesters. Due to various reasons, the expected grades in each subject if selected in different semesters, vary and they are given below.*

Semester	*Analysis*	*Statistics*	*Graph Theory*	*Algebra*
I	*F*	*E*	*D*	*C*
II	*E*	*E*	*C*	*C*
III	*C*	*D*	*C*	*A*
IV	*B*	*A*	*H*	*H*

The grade points are: $H = 10$, $A = 9$, $B = 8$, $C = 7$, $D = 6$, $E = 5$ and $F = 4$. How will the student select the electives in order to maximize the total expected points and what will be his maximum expected total points?

[Delhi B.Sc. (Stat.) 2

Solution. Since the given problem is to maximize the total expected points, we conver problem into that of minimization by subtracting all the elements of the grade points matrix fro highest grade point, *viz.*, *M* (= 10). The reduced matrix is as shown in table *11.13*.

Now subtract the minimum element of each row (column) from all the elements of the resp row (column) in table *11.13* and then make assignments in the rows and columns that have zeros. Thus, we have optimum table *11.14*

6	5	4	3
5	5	5	3
3	4	5	1
2	1	0	0

Table 11.13

1	1	1	$\boxed{0}$
$\cancel{0}$	1	$\boxed{0}$	$\cancel{0}$
$\boxed{0}$	2	2	$\cancel{0}$
$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

Table 11.14

From table *11.14*, we observe that the optimum assignment is to select :

I → Algebra, *II* → Graph Theory, *III* → Analysis and *IV* → Statistics.

Maximum expected total points will be *C* + *C* + *C* + *A*, *i.e.*, 3 × 7 + 9 or 30.

11.26. *The following is the cost matrix of assigning 4 clerks to 4 key punching jobs. Fin optimal assignment if clerk 1 cannot be assigned to job 1 :*

Clerk	*Job*			
	1	*2*	*3*	*4*
1	—	5	2	0
2	4	7	5	6
3	5	8	4	3
4	3	6	6	2

What is the minimum total cost? **[Dibrugarh M.Sc. (Stat.), 1994; C.A. Final (Nov.)**

Solution. Reduce the cost matrix by subtracting smallest element of each row (column) fro corresponding row (column) elements. In the reduced matrix make assignments in rows and co that have single zeros. Thus, we have :

Initial Iteration. Draw the minimum number of lines to cover all the zeros of the reduced m See table *11.15*.

Final Iteration. Modify the reduced cost matrix by subtracting element '1' from all the ele not covered by the lines and adding the same at the intersection of two lines. See table *11.16*.

M	2	1	$\boxed{0}$	✓
$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	2	
2	2	$\boxed{0}$	$\cancel{0}$	
1	1	3	$\cancel{0}$	✓
			✓	

Table 11.15

M	1	$\cancel{0}$	$\boxed{0}$
$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	3
2	2	$\boxed{0}$	1
$\cancel{0}$	$\boxed{0}$	2	$\cancel{0}$

Table 11.16

Since the number of assignments is equal to the order of the matrix, an optimum solu reached. Also, since there are at least two zeros for assignment in row 2 and row 4 as well column 1 and column 2, there exists an alternative assignment schedule. The optimum solution

Assign Clerk *1* to Job *4*, Clerk *2* to Job *1*, Clerk *3* to job *3* and Clerk *4* to Job *2*;

or Assign Clerk *1* to Job *4*, Clerk *2* to Job 2, Clerk *3* to Job 3 and Clerk *4* to Job *1*.

Total minimum cost will be 14.

Solution. Since the given problem is to maximize the total expected points, we conver problem into that of minimization by subtracting all the elements of the grade points matrix fro highest grade point, *viz.*, *W*|− 10). The reduced matrix is as shown in table *11.13*.

Now subtract the minimum element of each row (column) from all the elements of the resp row (column) in table *11.13* and then make assignments in the rows and columns that have zeros. Thus, we have optimum table *11.14*

6	5	4	3
5	5	5	3
3	4	5	1
2	1	0	0

Table 11.13

1	1	1	$\boxed{0}$
$\cancel{0}$	1	$\boxed{0}$	$\cancel{0}$
$\boxed{0}$	2	2	$\cancel{0}$
$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

Table 11.14

From table *11.14*, we observe that the optimum assignment is to select :

I → Algebra, *II* → Graph Theory, *III* → Analysis and *IV* → Statistics.

Maximum expected total points will be *C* + *C* + *C* + *A*, *i.e.*, 3 x 7 + 9 or 30.

11.26. *The following is the cost matrix of assigning 4 clerks to 4 key punching jobs. Fin optimal assignment if clerk 1 cannot be assigned to job 1 :*

Clerk	*Job*			
	1	*2*	*3*	*4*
1	—	5	2	0
2	4	7	5	6
3	5	8	4	3
4	3	6	6	2

What is the minimum total cost? *[Dibrugarh M.Sc. (Stat.), 1994; C.A. Final (Nov.)*

Solution. Reduce the cost matrix by subtracting smallest element of each row (column) fro corresponding row (column) elements. In the reduced matrix make assignments in rows and co that have single zeros. Thus, we have :

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Final Iteration. Modify the reduced cost matrix by subtracting element '1' from all the ele not covered by the lines and adding the same at the intersection of two lines. See table *11.16*.

M	2	1	$\boxed{0}$	√
$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	2	
2	2	$\boxed{0}$	$\cancel{0}$	
1	1	3	$\cancel{0}$	√
			√	

Table 11.15

M	1	$\cancel{0}$	$\boxed{0}$
$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	1
2	2	$\boxed{0}$	1
$\cancel{0}$	$\boxed{0}$	2	$\cancel{0}$

Table 11.16

Since the number of assignments is equal to the order of the matrix, an optimum solu reached. Also, since there are at least two zeros for assignment in row 2 and row 4 as well column 1 and column 2, there exists an alternative assignment schedule. The optimum solution

Assign Clerk *1* to Job *4*, Clerk *2* to Job *1*, Clerk *3* to job *3* and Clerk *4* to Job *2*;

or Assign Clerk *1* to Job *4*, Clerk *2* to Job 2, Clerk *3* to Job 3 and Clerk *4* to Job *1*.

Total minimum cost will be 14.

117. *In the modification of a plant layout of a factory, four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The cost of locating of machine i at place j (in hundred rupees) is shown below.*

				Location		
		A	*B*	*C*	*D*	*E*
Machine	M_1	9	11	15	10	11
	M_2	12	9	—	10	9
	M_3	—	11	14	11	7
	M_4	14	8	12	7	8

Find the optimal assignment schedule. ***[Delhi M.B.A. 2001]***

Solution. Since the number of machines ≠ number of vacant places, the cost matrix is unbalanced. So we add a dummy machine (row) with all its elements as zero. Also, assign a very high cost *M* to cells (3, 1) and (2, 3). The cost matrix so obtained is given in table *11.17*. Reduce this cost matrix by subtracting the smallest element of each row (column) from all the elements of that row (column). In the reduced matrix make assignments in rows and columns that have single zeros and cross off all other zeros of the row/column in which an assignment is made. Thus, we get table *11.18*.

	A	*B*	*C*	*D*	*E*
M_1	9	11	15	10	11
M_2	12	9	*M*	10	9
M_3	*M*	11	14	11	7
M_4	14	8	12	7	8
Dummy	0	0	0	0	0

Table 11.17

	A	*B*	*C*	*D*	*E*
M_1	$\boxed{0}$	2	6	1	2
M_2	3	$\boxed{0}$	*M*	1	~~0~~
M_3	*M*	4	7	4	$\boxed{0}$
M_4	7	1	5	$\boxed{0}$	1
Dummy	~~0~~	~~0~~	$\boxed{0}$	~~0~~	~~0~~

Table 11.18

Since the number of assignments in table *11.18* is equal to the order of the matrix, an optimum solution is reached. The optimum solution is :

Assign M_1 at *A*, M_2 at *B*, M_3 at *E* and M_4 at *D*, at the minimum total cost of Rs. 3,200.

PROBLEMS

118. M/S Inc is a software company that has three projects of Y2K with the departments of health, education and housing of Maharashtra Government. Based on the background and experiences of the project leaders, they differ in terms of their performance at various projects. The performance score matrix is given below:

Project leaders	*Projects*		
	Health	*Education*	*Housing*
P_1	20	36	42
P_2	24	32	50
P_3	32	34	44

Help the management by determining the optimal assignment that maximises the total performance score.

[Mumbai D.M.S. 1998]

119. Suggest the optimal assignment schedule for the following assignment problem:

Sales (Rs. in lakh)

Salesmen	*Markets*

11.17. *In the modification of a plant layout of a factory, four new machines M_1, M_2, M_3 and M_4 to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The cost of locating of machine i at place j (in hundred rupees) is shown below*

$$\begin{array}{ll|ccccc|}
 & & \multicolumn{5}{c}{\textit{Location}} \\
 & & A & B & C & D & E \\
\hline
 & M_1 & 9 & 11 & 15 & 10 & 11 \\
 & M_2 & 12 & 9 & - & 10 & 9 \\
\textit{Machine} & M_3 & - & 11 & 14 & 11 & 7 \\
 & M_4 & 14 & 8 & 12 & 7 & 8 \\
\hline
\end{array}$$

Find the optimal assignment schedule. ***[Delhi M.B.A. 2001]***

Solution. Since the number of machines ≠ number of vacant places, the cost matrix is unbalanced. So we add a dummy machine (row) with all its elements as zero. Also, assign a very high cost M to cells (3, 1) and (2, 3). The cost matrix so obtained is given in table *11.17*. Reduce this cost matrix by subtracting the smallest element of each row (column) from all the elements of that row (column). In the reduced matrix make assignments in rows and columns that have single zeros and cross off all other zeros of the row/column in which an assignment is made. Thus, we get table *11.18*.

$$\begin{array}{c|ccccc|}
 & A & B & C & D & E \\
\hline
M_1 & 9 & 11 & 15 & 10 & 11 \\
M_2 & 12 & 9 & M & 10 & 9 \\
M_3 & M & 11 & 14 & 11 & 7 \\
M_4 & 14 & 8 & 12 & 7 & 8 \\
\text{Dummy} & 0 & 0 & 0 & 0 & 0 \\
\hline
\end{array}$$

Table 11.17

$$\begin{array}{c|ccccc|}
 & A & B & C & D & E \\
\hline
M_1 & \boxed{0} & 2 & 6 & 1 & 2 \\
M_2 & 3 & \boxed{0} & M & 1 & \cancel{0} \\
M_3 & M & 4 & 7 & 4 & \boxed{0} \\
M_4 & 7 & 1 & 5 & \boxed{0} & 1 \\
\text{Dummy} & \cancel{0} & \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \\
\hline
\end{array}$$

Table 11.18

Since the number of assignments in table *11.18* is equal to the order of the matrix, an optimum solution is reached. The optimum solution is :

Assign M_1 at A, M_2 at B, M_3 at E and M_4 at D ; at the minimum total cost of Rs. 3,200.

PROBLEMS

11.18. M/S Inc is a software company that has three projects of Y2K with the departments of health, education and housing of Maharashtra Government. Based on the background and experiences of the project leaders, they differ in terms of their performance at various projects. The performance score matrix is given below

Project leaders	*Projects*		
	Health	*Education*	*Housing*
P_1	20	36	42
P_2	24	32	30
P_3	32	34	44

Help the management by determining the optimal assignment that maximises the total performance score.

[Mumbai D.M.S. 1998]

11.19. Suggest the optimal assignment schedule for the following assignment problem.

Sales (Rs. in lakh)

Salesmen	Markets

V - unit

Sequencing Problem

P-b-1

1. ~~t~~ A Company has 3 Jobs an hand Each of these must be processed through two departments the sequencial order for which is

Dept/A : Press shop.

Dept/B : Finishing

the table below lists the number of days required by each Job in each department.

	Job I	Job II	Job III
Dept A :	8	6	5
Dept B :	8	3	4.

Find the sequence in which the three Jobs should be processed so as to take minimum time to finish all the 3 Jobs.

Solu :-

Consider the number of days each Jobs take in the two department.

we see that min {AC, BC} = 3.

The only Corresponding to the Job as shown below

		II

the problem now reduces to the Following two Jobs in the two department

	Job I	Job II
Dept A :	8	5
Dept B :	8	4

Here min {AC, BC} = 4.

there fore the optial sequence is represented below

I	III	II

∴ the minimum elapsed time can this be computed cumulatively as Follows

machine A			machine B.	
Job	Time IN	Time out	Time in	Time out.
I	0	8	8	16
III	8	13	16	20
II	13	19	20	23

From the above table the minimum total time to finish all the 3 Jobs is 23 days.

Idle time For machine A = 23 - 19 = 4 days

Idle time For machine B = 8 + (16-16) + (20-20)

= 8 days.

12,03 we have Five Jobs, each of which must go through the two machines A and B in the order AB. processing times in hours are given in the table below.

Job(i) :	1	2	3	4	5
machine A(Ai) :	5	1	9	3	10
machine B(Bi) :	2	6	7	8	4.

Determine a sequence for the Five Jobs that will minimize the elapsed time.

Solu :-

Consider the number of time each Jobs take in the 2 machine

we see that min $\{A_i ; B_i\} = 1$

The entry corresponding to the Job as shown below

2				

the problem now reduces to the following two Jobs in the two machine.

Job	1	3	4	5
machine A	5	9	3	10
machine B	2	7	8	4

Here $\min_{i} \{A_i . B_i\} = 2$.

2				1

Job	3	4	5
machine A	9	3	10
machine B	7	8	4

Here $\min\{A_i\ B_i\} = 3$

2	4			1

Job	3	5
machine A	9	10
machine B	7	4

Here $\min_i = \{A_i, B_i\} = 3$

2	4	3	5	1

The minimum elapsed time can this be compted cumulatively as follows.

	machine A		machine B.	
Job	TIME IN	TIME OUT	TIME IN	TIME OUT
2	0	1	1	7
4	1	4	7	15
3	4	15	15	22
5	15	25	23	27
1	25	28	28	30.

From the above table the minimum total time to the finish all the 5 Jobs is 30 hours

Idle time For machine A = 30-28 = 2 hours

Idle time For machine B = 1+(7-7)+(15-15)+ (23-22)+(28-27)

= 1+1+1 = 3 hours.

---

3. Six Jobs go First over machine I and then over machine II - the order of the competetion of Jobs has no scientificance the Following table gives the machine times in hours For six Jobs and the two machine.

Job :	1	2	3	4	5	6
machine A :	5	9	4	7	8	6
machine II :	7	4	8	3	9	5

Find the sequence of Jobs that minimize the total elapsed time to complet the Jobs.

Solu :-

Consider the number of days each Jobs take in two machines.

$\min_i = \{A_i , B_i\} = 4$

					4

Job	1	2	3	4	5	6
machine I	5	9	4	8	8	6
machine II	7	4	8	9	5	

$\min i = \{Ai, Bi\} = 4$.

3	.	.	.	2	4

Job	1	5	6
machine I	5	8	6
machine II	7	9	5

$\min i = \{Ai, Bi\} = 5$.

8.

3	1	5	6	2	4

Job	machine I		machine II	
	time IN	time out	time in	time out
3	0	4	4	12
1	4	9	12	19
5	9		19	28
6	17	23	28	33
2	23	32	33	37
4	32	39	39	42

From the above table the minimum total hours to finish all the 6 Jobs in 42 hours

Idle hour for machine I $= 42 - 39$
$= 3$ hrs.

Idle hours for machine II $= 4 + (12-12) + (19-19) + (28-28) + (33-33) + (39-39)$
$= 6$ hrs.

1201 In a Factory there are six Jobs to perform, each of which should go through two machines A and B in the order A, B. The processing timings for the Jobs are given here. You are required to determine the sequence for performing the Jobs that would minimize the total elapsed time T. what is the value of T?

Job :	J_1	J_2	J_3	J_4	J_5	J_6
machine A :	1	3	8	5	6	3
machine B :	5	6	3	2	2	10

Solu^m :

Consider the number of days each Jobs take in the two department.

we see that $\min_i \{A_i \; B_i\} = 1$

J_1					

the problem now reduces to the following six Jobs in the two department

Job	:	J_2	J_3	J_4	J_5	J_6
machine A	:	3	8	5	6	3
machine B	:	6	3	2	2	10

$\min i \{A_i\ B_i\} = 2$

J_1				J_4	J_5

Job	:	J_2	J_3	J_6
machine A	:	3	8	3
machine B	:	6	3	10

$\min i \{A_i\ B_i\} = 3.$

J_1	J_6	J_2	J_3	J_4	J_5

the mimum elapsed time can this be completed cumulatively as Follows.

Job.	machine A		machine B	
	TIME IN	TIME OUT	TIME IN	TIME OUT
J_1	0			6
J_6		4	6	16
J_2	4	7	16	22
J_3	7	15	22	25
J_4	15	20	25	27
J_5	20	26	27	29

From the above table, the minimum time to finish all the 6 Jobs is 29 hrs.

Idle time For machine A = 29-26 = 3 hrs

Idle time for machine B = 1+(6-6)+(16-16)+

(22-22)+(25-25)+

=1 hrs.

1202 A book binder has one printing press, one binding machine and the manuscripts of a number of different books. The time required to perform the printing and binding operations for each book is shown below. Determine the order in which books should be processed in order to minimize the total time required to turn out all the books.

Book :	1	2	3	4	5	6
Printing time (hrs) :	30	120	50	20	90	100
Binding time (hrs) :	80	100	90	60	30	10.

Solu:

Consider the number of the each Books in the two hrs.

we see that min i {Ai Bi} = 10.

The entry corresponding to the books as shown below.

					6

Book :	1	2	3	4	5
P.T :	30	120	50	20	90
B.T :	80	100	90	60	30

$\min i \{A_i\ B_i\} = \{20\}$

Book :	1	2	3	5
P.T :	30	120	50	90
B.T :	80	100	90	30

4					6

$\min i \{A_i\ B_i\} = 30$

4	1			5	6

Book :	2	3
P.T :	120	50
B.T :	100	90

$\min i \{A_i\ B_i\} = 50$

4	1	3	2	5	6

Book	Printing time (hrs)		Binding time (hrs)	
	time IN	time out	time in	time out.
4		20	20	80
1	20	50	80	160
3	50	100	160	250
2	100	220	250	350
5	220	310	350	380
6	310	410	410	420

From the above table the minimum time to finish all the 6 books 420 hrs.

Idle time for printing time = 420 - 410
= 10 hrs.

Idle time for Binding time = 20 + (80 - 80) + (160 - 160) + (250 - 250) + (350 - 350) + (410 - 380)
= 50 hrs.

120T In the machine shop, 8 different products are being manufactured each requiring time on two machines A and B as given below:-

Products	I	II	III	IV	V	VI	VII	VIII
Time on machine A	30	45	15	20	80	120	65	10
Time on Machine B.	20	30	50	35	36	40	50	20

Decide the optimum sequence of processing of different products in order to minimize the total manufacting time for all the products. Name and discuss the scheduling model used.

Solu:-

Consider the number of days each Jobs take in the two machine

we see that min i = {10}

8							

Job	1	2	3	4	5	6	7
machine A	30	45	15	20	80	120	65
machine B.	20	30	50	35	36	40	50

min i = {Ai Bi} = 15

8	3						

Job	1	2	4	5	6	7
machine A	30	45	20	80	120	65
machine B	20	30	35	36	40	50

min i {Ai Bi} = 20

8	3	4					1

Job	2	5	6	7
machine A	45	80	120	65
machine B.	30	36	40	50

min i = {Ai Bi} = 30

8	3	4				2	1

Job	5	6	7
machine A	{80	120,	65
machine B	36	40	50

8

min i = {Ai Bi} = 36

8	3	4			5	2	1

Job : 6 7

machine A : 120 65

machine B : 40 50

min i = {Ai Bi} = 40

8	3	4	7	6	5	2	1

Job	machine A		machine B	
	TIME IN	TIME OUT	TIME IN	TIME OUT
8	0	10	10	30
3	10	25	30	80
4	25	45	80	115
7	45	110	115	165
6	110	230	230	270
5	230	310	310	346
2	310	355	355	385
1	355	385	385	405

From the take the minimum time to Finish all the Jobs is 405 hrs

Idle time for machine A = 405 - 385 = 20 hrs

Idle time for machine B = 10 + (30 - 30) + (80 - 80) + (115 + 115) + (230 - 165) + (310 + 270) + (355 - 346) + (385 - 385) = 124 hrs.

12(b) seven Jobs are to be processed on two machines A and B in the order A→B Each machine can to process only one Job at a time. The processing times are as follows.

Job :	1	2	3	4	5	6	7
machine A :	10	12	15	7	14	5	16
machine B :	15	11	8	9	6	7	16

Solu :-

Consider the number of two each Jobs take in the two machine.

we see $\min i = \{A_i \ B_i\} = 5$

6						

Job	1	2	3	4	5	7
machine A	10	12	15	7	14	16
machine B.	15	11	8	9	6	16

$\min i = \{A_i \ B_i\} = 6$

6						5

Job	1	2	3	4	7
machine A :	10	12	15	7	16
machine B :	15	11	8	9	16

$\min i = \{A_i \ B_i\} = 7$

6	4					5

Job : 1 2 3 7

machine A : 10 12 13 16

machine B: 15 11 8 16

min i = {Ai Bi} = 8

6	4				3	5

Job : 1 2 7

machine A : 10 12 16

machine B : 15 11 16

min i = {Ai Bi} = 10

6	4	1			3	5

Job : 2 7

machine A : 12 16

machine B : 11 16

min {Ai Bi} = 11

6	4	1	7	2	3	5

Job	machine A		machine B.	
	Time IN	Time out	Time in	Time out
6	0	5	5	12
4	5	12	12	21
1	12	22	22	37
7	22	38	38	54
2	38	50	54	65
3	50	63	65	73
5	63	77	77	83

From the given table the minimum time finish all the Jobs in 83 hrs.

Idle time for machine A = 83 - 77 = 6 hrs

Idle time for machine B = 3 + (12 - 12) + (22 - 21) + (38 - 37) + (54 - 54) + (65 - 6

+ (77 - 73)

= 11 hrs.

Processing n Jobs through k machines

12.09. Determine the optimal sequence of Jobs that minimizes the total elapsed time based on the following information processing time on machines is given in hours and passing is not allowed.

Job:	A	B	C	D	E	F	G
min machine M_1 :	3	8	7	4	9	8	7
max machine M_2 :	4	3	2	5	1	4	3
min machine M_3 :	6	7	5	11	5	6	12

Solu :-

Job	M_1	M_2	M_3
A	3	4	6
B	8	3	7
C	7	2	5
D	4	5	11
E	9	1	5
F	8	4	6
G	7	3	12

Here n=7 and K=3

K-1 = 3-1
= 2

we abserve, that

min tij = 3, max tij = 5.

min tij ≱ max tij , i=2,3 .... k-1

3 ≱ 5

min tij ≥ max tij is not satisfied

min ~~tij~~ t_{3j} = 5

max tij = 5

min t_{3j} ≥ max tij

5 ≥ 5

hence min tij ≥ max tij is satisfied

hence the problem can be converted into that Jobs 2 machiness.

Let G and H be the 2 machines such that

$G_i = t_{ij} + t_{2j}$ & $H_i = t_{2j} + t_{3j}$

Job	G	H
A	7	10
B	11	10
C	9	7
D	9	15
E	10	6
F	12	10
G	10	15

min i = {Ai Bi} = 6

						E

Job	A	B	C	D	F	G
G	7	11	9	9	12	10
H	10	10	7	15	10	15

min i = {Ai, Bi} = 7

A					C	E

Job	B	D	F	G
G	11	9	12	10
H	10	15	10	15

min i = {Ai Bi} = 9

A	D				C	E

Job	B	F	G
G	11	12	10
H	10	10	15

min i = {Ai Bi} = 10

A	D	G	B	F	C	E

To elapsed time we have

Job	M1			M2			M3		
	IN	OUT	Idle TIME	IN	OUT	Idle TIME	IN	OUT	Idle TIME
A	0	3	0	3	7	3	7	15	7
D	3	7	0	7	12	0	15	24	0
G	7	14	0	14	17	2	24	36	0
B	14	22	0	22	25	5	36	43	0
F	22	30	0	30	34	5	43	49	0
C	30	37	0	37	39	3	49	54	0
E	37	46	0	46	47	7	54	59	0
			0			25			7

The minimum total elapsed time is 59

Idle time of $M_1 = [0+(59-46)] = 13$ hours

Idle time of $M_2 = [25+(59-47)] = 37$

Idle time of $M_3 = 7$

1210 we have 4 Jobs each of which has to go through the machines M_j $(j=1,2 \ldots 6)$ in the order M_1 $M_2 \ldots M_6$ Precessing time is given below.

JOB	M_1	M_2	M_3	M_4	M_5	M_6
A	18	8	7	2	10	25
B	17	6	9	6	8	19
C	11	5	8	5	7	15
D	20	14	3	4	8	12

Determine a sequence of these four Jobs that minimizes the total elapsed time.

Solu :-

Job	M_1	M_2	M_3	M_4	M_5	M_6
A	18	8	7	2	10	25
B	17	6	9	6	8	19
C	11	5	8	5	7	15
D	20	4	3	4	8	12

$n=4, \ k=6$

we abserve that

$\min t_{ij} = 11, \ \max t_{ij} = 10$

$\min t_{ij} \geq \max t_{ij} \ , \ i=1,2 \ldots k-1$

$11 \geq 10$

$\min t_{ij} \geq \max t_{ij}$ is satisfied

$\min t_{6j} = 12$

$\max t_{ij} = 10$

$\min t_{6j} \geq \max t_{ij}$

the conditions $\min t_{6j} \geq \max t_{ij}$ is satisfied

The given problem can be written as

$$G_i = \sum_{i=1}^{5} t_{ij} \ \& \ H_i = \sum_{i=2}^{6} t_{ij}$$

Job	A	B	C	D
G	45	46	36	39
H	52	48	40	51

$\min i = 31$

			D

JOB A B C

G 45 46 36

H 52 48 40

min i = 36

C			D

JOB A B

G 45 46

H 52 48

min i = 45

C	A	B	D

Total elapsed time we have

Job	M_1			M_2			M_3			M_4			M_5			M_6.		
	IN	out	Idle T	IN	OUT	Idle T	IN	out	idle T	IN	out	Idle T	IN	OUT	Idle T	IN	OUT	Idle T
C	0	11	0	11	16	11	16	24	16	24	29	24	29	36	29	36	12	36
A	11	29	0	29	37	15	37	44	15	44	46	15	46	56	10	56	81	5
B	29	46	0	46	52	9	52	61	8	61	67	15	67	75	11	89	100	0
D	46	66	0	66	70	14	70	75	19	75	77	6	77	85	2	100	112	0
			0			47			46			60			52			41

The minimum total elapsed time is 112.

Idle time of $M_1 = [0+(112-66)] = 46$

Idle time of $M_2 = [47+(112-70)] =$

Idle time of $M_3 = [56+(112-73)] =$

Idle time of $M_4 = [66+(112-77)] =$

Idle time of $M_5 = [102+(112-85)] =$

Idle time of $M_6 = 41$

$$\begin{array}{r} 112 \\ -66 \\ \hline 46 \end{array}$$

12//. Solve the following sequencing problem when passing out is not allowed.

Item	A	B	C	D
I	15	5	4	15
II	12	2	10	12
III	16	3	5	16
IV	17	3	4	17

Solu:-

Item	A	B	C	D
I	15	5	4	15
II	12	2	10	12
III	16	3	5	16
IV	17	3	4	17

$n=4$, $K=4$

we abserve that,

$\min t_{ij} \geq \max t_{ij}$

$12 \geq 10$

$\min t_{ij} \geq \max t_{ij}$ is satisfied.

$\min t_{4j} = 12$, $\max t_{ij} = 10$

$\min t_{4j} \geq \max t_{ij}$

$12 \geq 10$

$\min t_{4j} \geq \max t_{ij}$ is satisfied.

The given problem can be written

$$G_i = \sum_{i=1}^{3} t_{ij} \ \& \ H_i \sum_{i=2}^{4} t_{4j}$$

Item	I	II	III	IV
G	24	24	24	24
H	24	24	24	24.

min $i = 24$

I	II	III	IV

Item	A			B			C			D		
	IN	OUT	Idle	IN	OUT	Idle	IN	out	Idle	IN	out	Idl
I	0	15	0	15	20	15	20	24	20	24	39	24
II	15	27	0	27	29	7	29	39	5	39	51	0
III	27	43	0	43	46	14	46	51	7	51	67	0
IV	43	60	0	60	65	14	65	67	12	67	84	0
			0			50			44			

The minimum total elapsed time is = 84

Idle time For A = 84 - 60 = 24 hrs

Idle time For B = 50 + (84 - 65) = 71 hrs

Idle time For c = 44 + (84 - 67) = 61 hrs

Idle time For D = 24 hrs.

---

1. we have Five Jobs each of which must go through machine A, B and c in the order ABC Precessing times are given in the Following table.

Job	1	2	3	4	5
A	8	10	6	7	11
B	5	6	2	3	4
c	4	9	8	6	5

Solu :-

Job : ~~A~~	1	2	3	4	5
machine A :	8	10	6	7	11
machine B :	5	6	2	3	4
machine C :	4	9	8	6	5

Hence $n = 5$, $k = 3$.

$\min t_{ij} = 6$, $\max t_{ij} = 6$

$\min t_{ij} \geq \max t_{ij}$

$\min t_{3j} = 4, \quad \max t_{ij} = 6$

$4 \neq 6$

Let G and H be the two machine such that

$G_i = t_{1j} + t_{2j}, \quad H_i = t_{2j} + t_{3j}$

Job	1	2	3	4	5
G	13	16	8	10	15
H	9	15	10	9	9

The optimal sequence is

3	2	4	1	5

The total elapsed time

Job	A			B			C		
	IN	OUT	Idle Time	IN	OUT	Idle Time	IN	OUT	Id Ti
3	0	6	0	6	8	6	8	16	8
2	6	16	0	16	22	8	22	31	6
4	16	23	0	23	26	1	31	37	0
1	23	31	0	31	36	5	37	41	0
5	31	42	0	42	46	6	46	51	5
			0			26			1

The minimum total elapsed time is 51 hrs.

Idle time A = 0 + (51 − 42) = 9

Idle time of B = 26 + (51 − 46) = 31

Idle time of C = 19.

1218. Solve the following sequencing problem, giving an opti solution when passing is not allowed.

Job	:	A	B	C	D	E
machine M_1	:	10	12	8	15	16
machine M_2	:	3	2	4	1	5
machine M_3	:	5	6	4	7	3
machine M_4	:	14	7	12	8	10

solu :-

Job	:	A	B	C	D	E
M_1	:	10	12	8	15	16
M_2	:	3	2	4	1	5
M_3	:	5	6	4	7	3
M_4	:	14	7	12	8	10

Here $n=5$, $k=4$

min $t_{ij}=8$, max $t_{ij}=7$

min $t_{ij} \geq$ max t_{ij}

$8 \geq 7$ is satisfied

min $t_{4j}=7$, max $t_{ij}=7$

$7 \geq 7$ is satisfied

The given problem can be written as

$G_i = t_{1j} + t_{2j} + t_{3j}$

$H_i = t_{2j} + t_{3j} + t_{4j}$

Job	A	B	C	D	E
G	18	20	16	23	24
H	22	15	20	16	18

The optimal sequence is

C	A	E	D	B

The total elapsed time.

Job	M_1			M_2			M_3			M_4		
	IN	OUT	idle	IN	OUT	idle	IN	OUT	Idle	IN	OUT	Idl
C	0	8	0	8	12	8	12	16	12	16	28	16
A	8	18	0	18	21	6	21	26	5	28	42	0
E	18	34	0	34	39	13	39	42	13	42	52	0
D	34	49	0	49	50	10	50	57	8	57	65	5
B	49	61	0	61	63	11	63	69	6	69	76	4
			0			48			44			25

The minimum total elapsed time is 76 hrs

Idle time for $M_1 = 0+(76-61) = 15$ hrs

Idle time for $M_2 = 48+(76-63) = 61$ hrs

Idle time for $M_3 = 44+(76-69) = 51$ hrs

Idle time for $M_4 = 25$ hrs.

1220 when passing is not allowed solve the sequencing Prob giving an optimal solution:

Job	:	A	B	C	D
Machine M_1	:	11	8	12	15
Machine M_2	:	4	1	2	2
Machine M_3	:	2	8	5	
Machine M_4	:	11	8	12	15

Solu :-

Here $n=4, K=4$

$\min t_{ij} = 8, \quad \max t_{ij} = 8$

$\min t_{ij} \geq \max t_{ij}$ is satisfied

$G_i = \sum_{i=1}^{3} t_i, \quad H_i = \sum_{2}^{4} t_i$

Job	A	B	C	D
G	17	17	17	17
H	17	17	17	17

The optimal sequence is

A	B	C	D

The total elapsed time.

Job	M_1			M_2			M_3			M_4		
	IN	OUT	Idle Time	IN	OUT	Idle	IN	OUT	Idle	IN	OUT	Idle
A	0	11	0	11	15	11	45	17	15	17	28	17
B	11	19	0	19	20	4	20	28	3	28	36	0
C	19	31	0	31	33	11	35	36	5	36	48	0
D	31	44	0	44	46	11	46	48	10	48	61	0
			0			37			35			17

The minimum total elapsed time is 61 hrs

Idle time for $M_1 = 61-44 = ~~0~~ 17$

Idle time for $M_2 = ~~6~~ 37+(61-46) = 52$

Idle time for $M_3 = 35+(61-48) = 46$

Idle time for $M_4 = 17$.

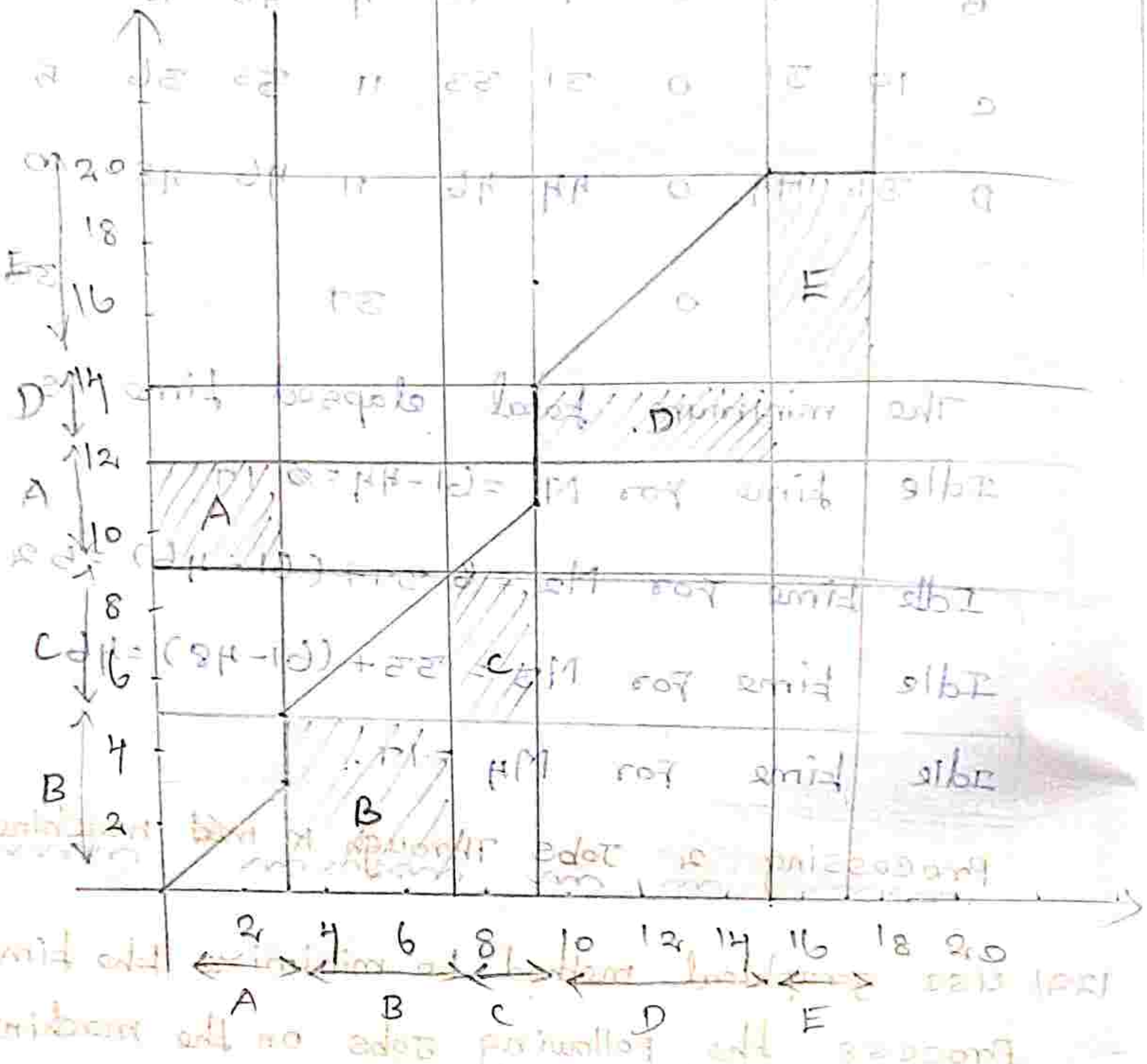
Processing 2 Jobs Through K ~~mcd~~ machines

12.21. Use graphical method to minimize the time added to process the following jobs on the machines shown i.e for each machine find the job which should be done first. Also calculate the total time elapsed to complete both the jobs.

Job 1	sequence	A	B	C	D	E
	Time.	3	4	2	6	2
Job 2	sequence	B	C	A	D	E
	Time	5	4	3	2	6

Solution :-

Draw two axes at right angle to each other which x-axis represents the processing time of Job 1 on different machine while Job 2 remains Idle.



Idle time for Job 1 = 17+2+3

= 22

Idle time for Job 2 = 20+2

= 22.